1.10 – Finding Limits

There are three (3) ways to find a limit: numerically, graphically, and analytically.
Numerically – by using a table of values
Graphically – self explanatory
Analytically – by using algebra, most commonly through substitution
Let's go over some examples and explore these methods ☺

Example 1 Find the limit of $f(x) = \frac{x}{\sqrt{x+1}-1}$ numerically. Then confirm graphically and analytically

$$\lim_{x \to 0} \frac{x}{\sqrt{x+1}-1} =$$

x	-0.25	-0.1	-0.01	-0.001	0	0.001	0.01	0.1	0.25
y	1.866	1.949	1.995	1.999	UND	2.0005	2.005	2.049	2.118

→ 2 ←

a) Numerically → 2

b) Graphically

c) Analytically ① plug in x = 0

$f(0) = \frac{0}{\sqrt{0+1}-1} = \frac{0}{\sqrt{1}-1} = \frac{0}{0} =$ UND Failed

② Simplify, then plug in (multiply conjugate)

$$\frac{x}{\sqrt{x+1}-1}\left(\frac{\sqrt{x+1}+1}{\sqrt{x+1}+1}\right) = \frac{x\sqrt{x+1}+x}{(x+1)+\sqrt{x+1}-\sqrt{x+1}-1} = \frac{x\sqrt{x+1}+x}{x+1-1}$$

$$= \frac{x\sqrt{x+1}+x}{x} = \sqrt{x+1}+1$$

$$f(0) \Rightarrow \lim_{x\to 0} \sqrt{x+1}+1 = 2$$

HW: 1-20

In Exercises 1–8, complete the table and use the result to estimate the limit. Use a graphing utility to graph the function to confirm your result.

1. $\lim_{x \to 2} \frac{x - 2}{x^2 - x - 2}$

x	1.9	1.99	1.999	2.001	2.01	2.1
$f(x)$						

2. $\lim_{x \to 2} \frac{x - 2}{x^2 - 4}$

x	1.9	1.99	1.999	2.001	2.01	2.1
$f(x)$						

3. $\lim_{x \to 0} \frac{\sqrt{x + 3} - \sqrt{3}}{x}$

x	−0.1	−0.01	−0.001	0.001	0.01	0.1
$f(x)$						

4. $\lim_{x \to -3} \frac{\sqrt{1 - x} - 2}{x + 3}$

x	−3.1	−3.01	−3.001	−2.999	−2.99	−2.9
$f(x)$						

5. $\lim_{x \to 3} \frac{[1/(x + 1)] - (1/4)}{x - 3}$

x	2.9	2.99	2.999	3.001	3.01	3.1
$f(x)$						

6. $\lim_{x \to 4} \frac{[x/(x + 1)] - (4/5)}{x - 4}$

x	3.9	3.99	3.999	4.001	4.01	4.1
$f(x)$						

7. $\lim_{x \to 0} \frac{\sin x}{x}$

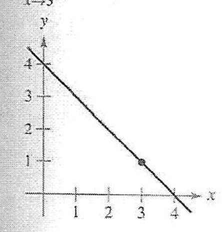
x	−0.1	−0.01	−0.001	0.001	0.01	0.1
$f(x)$						

8. $\lim_{x \to 0} \frac{\cos x - 1}{x}$

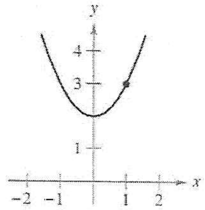
x	−0.1	−0.01	−0.001	0.001	0.01	0.1
$f(x)$						

In Exercises 9–18, use the graph to find the limit (if it exists). If the limit does not exist, explain why.

9. $\lim_{x \to 3} (4 - x)$

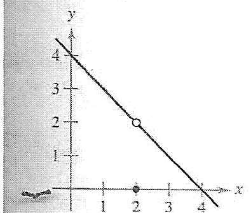


10. $\lim_{x \to 1} (x^2 + 2)$



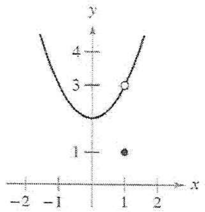
11. $\lim_{x \to 2} f(x)$

$$f(x) = \begin{cases} 4 - x, & x \neq 2 \\ 0, & x = 2 \end{cases}$$

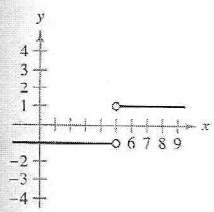


12. $\lim_{x \to 1} f(x)$

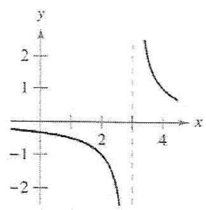
$$f(x) = \begin{cases} x^2 + 2, & x \neq 1 \\ 1, & x = 1 \end{cases}$$



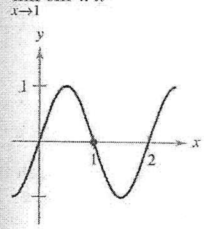
13. $\lim_{x \to 5} \frac{|x - 5|}{x - 5}$



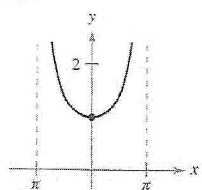
14. $\lim_{x \to 3} \frac{1}{x - 3}$



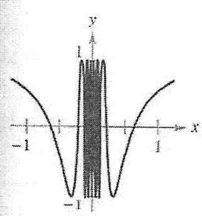
15. $\lim_{x \to 1} \sin \pi x$



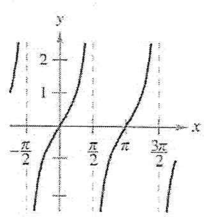
16. $\lim_{x \to 0} \sec x$



17. $\lim_{x \to 0} \cos \frac{1}{x}$

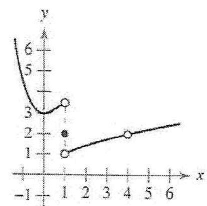


18. $\lim_{x \to \pi/2} \tan x$

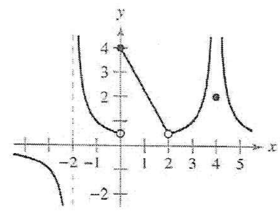


In Exercises 19 and 20, use the graph of the function f to decide whether the value of the given quantity exists. If it does, find it. If not, explain why.

19. (a) $f(1)$
(b) $\lim_{x \to 1} f(x)$
(c) $f(4)$
(d) $\lim_{x \to 4} f(x)$

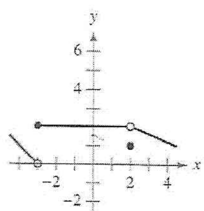


20. (a) $f(-2)$
(b) $\lim_{x \to -2} f(x)$
(c) $f(0)$
(d) $\lim_{x \to 0} f(x)$
(e) $f(2)$
(f) $\lim_{x \to 2} f(x)$
(g) $f(4)$
(h) $\lim_{x \to 4} f(x)$

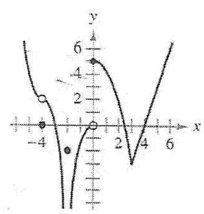


In Exercises 21 and 22, use the graph of f to identify the values of c for which $\lim_{x \to c} f(x)$ exists.

21.



22.



In Exercises 23 and 24, sketch the graph of f. Then identify the values of c for which $\lim_{x \to c} f(x)$ exists.

23. $f(x) = \begin{cases} x^2, & x \leq 2 \\ 8 - 2x, & 2 < x < 4 \\ 4, & x \geq 4 \end{cases}$

24. $f(x) = \begin{cases} \sin x, & x < 0 \\ 1 - \cos x, & 0 \leq x \leq \pi \\ \cos x, & x > \pi \end{cases}$