1.2 - Polynomial Zeros and Extrema

Zeros of a polynomial – x-intercepts of the graph of a polynomial function

Extrema of a polynomial – where the graph changes from increasing to decreasing or vice versa

A Polynomial function f of degree d, has at most d zeros and at most d - 1 extrema

Example 1 State the possible number of real zeros and extrema

$f(x) = x^3 - 5x^2 + 6x$

$d = 3$ zeros

$d - 1 = 2$ extrema

Factoring Polynomials

Quadratic Form

Words	A polynomial expression in x is in **quadratic form** if it is written as $au^2 + bu + c$ for any numbers a, b, and c, $a \neq 0$, where u is some expression in x.
Example	$x^4 - 5x^2 - 14$ is in quadratic form because the expression can be written as $(x^2)^2 - 5(x^2) - 14$. If $u = x^2$, then the expression becomes $u^2 - 5u - 14$.

Example 2 State the number of real zeros and extrema, then determine all real zeros.

a. $f(x) = x^4 - 3x^2 - 4$ $d = 4$ at most 4 zeros 3 extrema

$0 = (x^2)^2 - 3(x^2) - 4$

$0 =$ let $(x^2) = u$

$0 = u^2 - 3u - 4$

$0 = (u+1)(u-4) = (x^2+1)(x^2-4) = (x^2+1)(x+2)(x-2)$

$x^2 + 1 = 0$, Not real $x + 2 = 0$ $x - 2 = 0$ $\boxed{x = \pm 2}$

2 zeros
3 extrema

b. $f(x) = x^5 - 6x^3 - 16x$

$0 = x(x^4 - 6x^2 - 16)$

$0 = x[(x^2)^2 - 6(x^2) - 16]$

$0 = x(u^2 - 6u - 16)$

$0 = x(u-8)(u+2)$

$0 = x(x^2 - 8)(x^2 + 2)$

$\boxed{x = 0}$ $\boxed{x = \pm\sqrt{8}}$ Not real

3 zeros
2 extrema @ most 4

HW: 23-32

State the number of possible real zeros and turning points of each function. Then determine all of the real zeros by factoring. (Examples 3–5)

23. $f(x) = x^5 + 3x^4 + 2x^3$

24. $f(x) = x^6 - 8x^5 + 12x^4$

25. $f(x) = x^4 + 4x^2 - 21$

26. $f(x) = x^4 - 4x^3 - 32x^2$

27. $f(x) = x^6 - 6x^3 - 16$

28. $f(x) = 4x^8 + 16x^4 + 12$

29. $f(x) = 9x^6 - 36x^4$

30. $f(x) = 6x^5 - 150x^3$

31. $f(x) = 4x^4 - 4x^3 - 3x^2$

32. $f(x) = 3x^5 + 11x^4 - 20x^3$