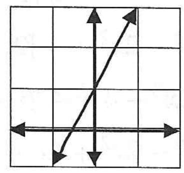
1.5 - Average Rates of Change

If you recall, the formula to find the slope of a line is:

$m = \frac{y_2 - y_1}{x_2 - x_1}$. For the line at the right, find the slope using the points (-1, -1) and (1, 3).



$m = \frac{3-(-1)}{1-(-1)} = \frac{4}{2} = 2$

The slope of a line is constant and therefore represents a constant rate of change. The slope of a curve however, is always changing. For this reason, we look for an average rate of change.

So the average rate of change of a curve can be represented as:

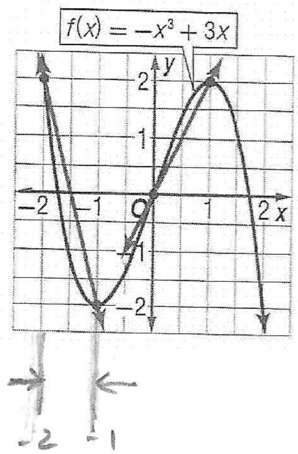
$$m_{ave} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Example 1 Find the average rate of change of $f(x) = -x^3 + 3x$ on each interval.

a. [-2, -1]

x_1, x_2

$M_{ave} = \frac{f(-1) - f(-2)}{(-1) - (-2)}$



$M_{ave} = \frac{[-(-1)^3 + 3(-1)] - [-(-2)^3 + 3(-2)]}{1}$

$= -2 - 2$

$= -4$

b. [0, 1]

x_1, x_2

$M_{ave} = \frac{f(1) - f(0)}{1 - 0}$

$M_{ave} = \frac{[-(1)^3 + 3(1)] - [-(0)^3 + 3(0)]}{1}$

$= 2 - 0$

$= 2$

Find the average rate of change of each function on the given interval. (Example 5)

34. $g(x) = -4x^2 + 3x - 4; [-1, 3]$

35. $g(x) = 3x^2 - 8x + 2; [4, 8]$

36. $f(x) = 3x^3 - 2x^2 + 6; [2, 6]$

37. $f(x) = -2x^3 - 4x^2 + 2x - 8; [-2, 3]$

38. $f(x) = 3x^4 - 2x^2 + 6x - 1; [5, 9]$

39. $f(x) = -2x^4 - 5x^3 + 4x - 6; [-1, 5]$

40. $h(x) = -x^5 - 5x^2 + 6x - 9; [3, 6]$

41. $h(x) = x^5 + 2x^4 + 3x - 12; [-5, -1]$

42. $f(x) = \frac{x-3}{x}; [5, 12]$

43. $f(x) = \frac{x+5}{x-4}; [-6, 2]$

44. $f(x) = \sqrt{x+8}; [-4, 4]$

45. $f(x) = \sqrt{x-6}; [8, 16]$