1.6 - Continuity and Limits

Example 1 Limits

$f(x) = -2x + 4$

Find $\lim_{x \to 1} f(x)$ $c = 1$

approaching → ↓ ← approaching

x	.9	.99	.999	1	1.001	1.01	1.1
$f(x)$	2.2	2.02	2.002	2	1.998	1.98	1.8

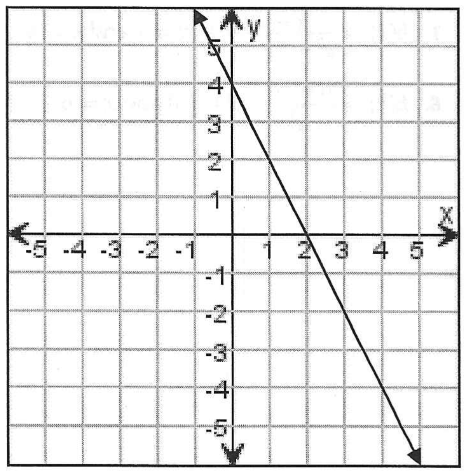
approaching → ↑ ← approaching

$\lim_{x \to 1} f(x) = 2$

Key Concept **Limits**

Words If the value of $f(x)$ approaches a unique value L as x approaches c from each side, then the limit of $f(x)$ as x approaches c is L.

Symbols $\lim_{x \to c} f(x) = L$, which is read *The limit of f(x) as x approaches c is L.*



Continuity of a function – if the graph of a function has no breaks, holes, or gaps it is said to be continuous. You can trace the graph of a continuous function without lifting your pencil

Concept Summary **Continuity Test**

A function $f(x)$ is continuous at $x = c$ if it satisfies the following conditions.

1. • $f(x)$ is defined at c. That is, $f(c)$ exists.
2. • $f(x)$ approaches the same value from either side of c. That is, $\lim_{x \to c} f(x)$ exists.
3. • The value that $f(x)$ approaches from each side of c is $f(c)$. That is, $\lim_{x \to c} f(x) = f(c)$.

Example 2 $f(x) = 2x^2 - 3x - 1 ; x = 2$

① $f(2) = 2(2)^2 - 3(2) - 1 \Rightarrow f(2) = 1$ ✓
$= 1$

②

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$f(x)$	.52	.9502	.995	1	1.005	1.05	1.52

$\Rightarrow \lim_{x \to 2} f(x) = 1$ ✓

③ $f(2) = \lim_{x \to 2} f(x)$, that is step ① = step ②

Determine whether each function is continuous at the given x-value(s). Justify using the continuity test.

1. $f(x) = \sqrt{x^2 - 4}$; at $x = -5$
2. $f(x) = \sqrt{x + 5}$; at $x = 8$
3. $h(x) = \frac{x^2 - 36}{x + 6}$; at $x = -6$ and $x = 6$
4. $h(x) = \frac{x^2 - 25}{x + 5}$; at $x = -5$ and $x = 5$
5. $g(x) = \frac{x}{x - 1}$; at $x = 1$
6. $g(x) = \frac{2 - x}{2 + x}$; at $x = -2$ and $x = 2$
7. $h(x) = \frac{x - 4}{x^2 - 5x + 4}$; at $x = 1$ and $x = 4$
8. $h(x) = \frac{x(x - 6)}{x^3}$; at $x = 0$ and $x = 6$