

1.7 - Rational Functions

Rational Function – a quotient of two polynomial functions $a(x)$ and $b(x)$ where $b(x) \neq 0$

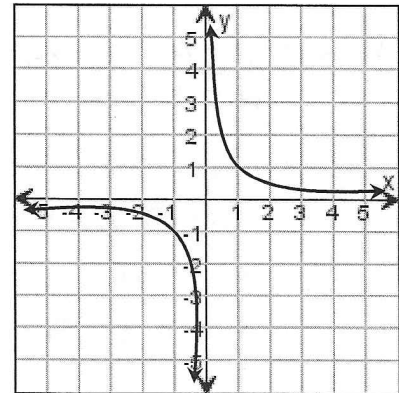
$$f(x) = \frac{a(x)}{b(x)}, \quad b(x) \neq 0$$

The DOMAIN of a rational function is $\{R\}$ excluding those values that make the denominator 0

The Reciprocal Function is one of the simplest forms of a rational function. $f(x) = \frac{1}{x}$

From the graph we see that it has branches, (like many rational functions) that approach specific x - and y -values

The lines that represent these values are called ASYMPTOTES



Horizontal and Vertical Asymptotes for $f(x) = \frac{a(x)}{b(x)}$

- ❖ Vertical asymptotes may occur at the real zeros of $b(x)$
- ❖ Horizontal asymptotes (let $a(x)$ be of a degree n and let $b(x)$ be of degree m)
 - if $n < m$, the horizontal asymptote is $y = 0$
 - if $n = m$, the horizontal asymptote is the ratio of leading coefficients of $\frac{a(x)}{b(x)}$
 - if $n > m$, there is no horizontal asymptote

x -intercepts – real zeros of $a(x)$

y -intercept - $f(0)$

Example 1 Find the Domain, the equations of any vertical and horizontal asymptotes, and the x - and y -intercepts

$$f(x) = \frac{6}{x+3}$$

Domain: $\{x \mid x \neq -3, x \in R\}$

x -intercepts: NONE

y -intercept: $x = 0$

$$y = \frac{6}{0+3} = 2$$

Vertical Asymptote: $x+3=0$
 $x = -3$

Horizontal Asymptote: $\frac{n=0}{m=1} \quad n < m$
 $y = 0$

Example 2 Find the Domain, the equations of any vertical and horizontal asymptotes, and the x- and y-intercepts

$$f(x) = \frac{x^2 - 7x + 10}{x - 3}$$

$$d: \begin{aligned} x - 3 &= 0 \\ x &= 3 \end{aligned}$$

$$D: \{x \mid x \neq 3, x \in \mathbb{R}\}$$

$$V_{asy}: \begin{aligned} x - 3 &= 0 \\ x &= 3 \end{aligned}$$

$$H_{asy}: \frac{n=2}{m=1} \quad n > m \quad \boxed{\text{None}}$$

$$\begin{aligned} X\text{-int}: & \quad x^2 - 7x + 10 \\ & \quad (x-5)(x-2) \\ & \quad x-5=0 \quad x-2=0 \\ & \quad \boxed{x=2, 5} \end{aligned}$$

$$y\text{-int} = y = \frac{0^2 - 7(0) + 10}{0 - 3} = \boxed{\frac{10}{-3}}$$

HW: ON A SEPARATE SHEET OF PAPER

For each function, determine any asymptotes and intercepts, and state the domain

$$1. f(x) = \frac{x-6}{x^2+8x+15}$$

$$2. f(x) = \frac{x^2+9x+20}{x-4}$$

$$3. f(x) = \frac{x^2-2x-8}{x^2-2x-3}$$