3.2 Solving Linear Systems Algebraically

There are 2 methods for solving linear systems. The substitution method and by elimination.

The Substitution Method

Step 1	Solve one of the equations for one of its variables
Step 2	Substitute the expression from Step 1 into the other equation and solve for the other variable
Step 3	Substitute the value from Step 2 into the revised equation from Step 1 and solve

Example 1 Solve the system using substitution

$$2x + 5y = -5$$
$$x + 3y = 3$$

$\boxed{S1}$ $x + 3y = 3$

$-3y \quad -3y$

$x = -3y + 3$

$\Downarrow$

$\boxed{S2}$ $2(-3y+3) + 5y = -5$

$-6y + 6 + 5y = -5$

$\quad -6 \quad\quad -6$

$-y = -11$

$\boxed{y = 11}$

$\boxed{S3}$ $x = -3(11) + 3$

$x = -33 + 3$

$\boxed{x = -30}$

$\Rightarrow$ Check $2(-30) + 5(11) = -5$ ✓

$(-30) + 3(11) = 3$ ✓

The Elimination Method

Step 1	Multiply one or both of the equations by a constant to obtain coefficients that differ only in sign for one of the variables
Step 2	Add the revised equations from Step 1. Combining like terms will eliminate one of the variables. Solve for the remaining variable
Step 3	Substitute the value obtained in Step 2 into either of the original equations and solve for the other variable.

Example 2 Solve the system using the elimination method

$$3x - 7y = 10$$
$$6x - 8y = 8$$

$\boxed{S1}$ $(3x - 7y = 10)(-2)$

$[-6x + 14y = -20]$

$\boxed{S2}$ $-6x + 14y = -20$

$+ \quad 6x - 8y = 8$

$6y = -12$

$\boxed{y = -2}$

$\boxed{S3}$ $3x - 7(-2) = 10$

$3x + 14 = 10$

$\quad -14 \quad\quad -14$

$3x = -4$

$\boxed{x = -\frac{4}{3}}$

Practice:

$4x + 3y = -2$

S1 $x + 5y = -9$

$-5y \quad -5y$

$x = -5y - 9$

S2 $4(-5y-9) + 3y = -2$

$-20y - 36 + 3y = -2$

$+36 \qquad +36$

$-17y = 34$

$\boxed{y = -2}$

S3 $x = -5(-2) - 9$

$x = 10 - 9$

$\boxed{x = 1}$

S1 $3x + 3y = -15$ (3)

$5x - 9y = 3$

S2 $9x + 9y = -45$

$+5x - 9y = 3$

$14x = -42$

$\boxed{x = -3}$

S3 $3(-3) + 3y = -15$

$-9 + 3y = -15$

$3y = -6$

$\boxed{y = -2}$

HW: 3, 4, 16, 17

EXAMPLES 1 and 4 on pp. 160–163 for Exs. 3–14

SUBSTITUTION METHOD **Solve the system using the substitution method.**

3. $2x + 5y = 7$, $x + 4y = 2$ (6, −1)

4. $3x + y = 16$, $2x - 3y = -4$ (4, 4)

5. $6x - 2y = 5$, $-3x + y = 7$

6. $x + 4y = 1$, $3x + 2y = -12$

7. $3x - y = 2$, $6x + 3y = 14$

8. $3x - 4y = -5$, $-x + 3y = -5$

9. $3x + 2y = 6$, $x - 4y = -12$

10. $6x - 3y = 15$, $-2x + y = -5$

11. $3x + y = -1$, $2x + 3y = 18$

12. $2x - y = 1$, $8x + 4y = 6$

13. $3x + 7y = 13$, $x + 3y = -7$

14. $2x + 5y = 10$, $-3x + y = 36$

EXAMPLES 2 and 4 on pp. 161–163 for Exs. 15–27

ELIMINATION METHOD **Solve the system using the elimination method.**

15. $2x + 6y = 17$, $2x - 10y = 9$

16. $4x - 2y = -16$, $-3x + 4y = 12$ (−4, 0)

17. $3x - 4y = -10$, $6x + 3y = -42$ (6, −2)

18. $4x - 3y = 10$, $8x - 6y = 20$

19. $5x - 3y = -3$, $2x + 6y = 0$

20. $10x - 2y = 16$, $5x + 3y = -12$

21. $2x + 5y = 14$, $3x - 2y = -36$

22. $7x + 2y = 11$, $-2x + 3y = 29$

23. $3x + 4y = 18$, $6x + 8y = 18$

24. $2x + 5y = 13$, $6x + 2y = -13$

25. $4x - 5y = 13$, $6x + 2y = 48$

26. $6x - 4y = 14$, $2x + 8y = 21$