4.6 – Operations with Complex Numbers

The Imaginary Unit i.

$i = \sqrt{-1}$ **therefore,** $i^2 = (\sqrt{-1})(\sqrt{-1}) = (\sqrt{-1})^2 = -1$

KEY CONCEPT *For Your Notebook*

The Square Root of a Negative Number

Property	Example
1. If r is a positive real number, then $\sqrt{-r} = i\sqrt{r}$.	$\sqrt{-3} = i\sqrt{3}$
2. By Property (1), it follows that $(i\sqrt{r})^2 = -r$.	$(i\sqrt{3})^2 = i^2 \cdot 3 = -3$

Example 1 Solve $2x^2 + 11 = -37$

$-11 \quad -11$

$\frac{2x^2}{2} = \frac{-48}{2}$

$x^2 = -24$

$x = \sqrt{-1}\sqrt{4}\sqrt{6} \Rightarrow \boxed{x = \pm 2i\sqrt{6}}$

Complex Numbers – written in Standard Form $a + bi$, consist of a real part, a, and an imaginary part, bi

To add and subtract complex numbers, combine the real parts and then combine the imaginary parts

Example 2 Write the expression as a complex number in standard form

a. $(8 - i) + (5 + 4i)$

$(8 + 5) + (-i + 4i)$

$\boxed{13 + 3i}$

b. $(7 - 6i) - (3 - 6i)$

$(7 - 3) + (-6i + 6i)$

$4 + 0i \quad \boxed{= 4}$

c. $10 - (6 + 7i) + 4i$

$(10 - 6) + (-7i + 4i)$

$4 + (-3i)$

$\boxed{4 - 3i}$

Example 3 Multiply Complex Numbers

a. $4i(-6 + i)$

$4i(-6) + 4i(i)$

$= -24i + 4i^2$

$= -24i + 4(-1)$

$= \boxed{-4 - 24i}$

b. $(9 - 2i)(-4 + 7i)$

$-36 + 63i + 8i - 14i^2$

$-36 - 14(-1) + 71i$

$= \boxed{-22 + 71i}$

Example 4 Divide Complex Numbers

Use Conjugate $(1+4i)$ & FOIL

Write the quotient $\frac{7+5i}{1-4i}$ in standard form

$$\frac{7+5i}{1-4i}\left(\frac{1+4i}{1+4i}\right) = \frac{7+28i+5i+20i^2}{1+4i-4i-16i^2}$$

$$= \frac{7+20(-1)+33i}{1-16(-1)} = \frac{-13+33i}{17} \text{ OR } -\frac{13}{17} + \frac{33i}{17}$$

HW: 3-33 all

EXAMPLE 1
on p. 275
for Exs. 3–11

SOLVING QUADRATIC EQUATIONS **Solve the equation.**

3. $x^2 = -28$
4. $r^2 = -624$
5. $z^2 + 8 = 4$
6. $s^2 - 22 = -112$
7. $2x^2 + 31 = 9$
8. $9 - 4y^2 = 57$
9. $6t^2 + 5 = 2t^2 + 1$
10. $3p^2 + 7 = -9p^2 + 4$
11. $-5(n-3)^2 = 10$

EXAMPLE 2
on p. 276
for Exs. 12–21

ADDING AND SUBTRACTING **Write the expression as a complex number in standard form.**

12. $(6 - 3i) + (5 + 4i)$
13. $(9 + 8i) + (8 - 9i)$
14. $(-2 - 6i) - (4 - 6i)$
15. $(-1 + i) - (7 - 5i)$
16. $(8 + 20i) - (-8 + 12i)$
17. $(8 - 5i) - (-11 + 4i)$
18. $(10 - 2i) + (-11 - 7i)$
19. $(14 + 3i) + (7 + 6i)$
20. $(-1 + 4i) + (-9 - 2i)$

21. ★ **MULTIPLE CHOICE** What is the standard form of the expression $(2 + 3i) - (7 + 4i)$?

(A) -4 (B) $-5 + 7i$ (C) $-5 - i$ (D) $5 + i$

EXAMPLES 4 and 5
on pp. 277–278
for Exs. 22–33

MULTIPLYING AND DIVIDING **Write the expression as a complex number in standard form.**

22. $6i(3 + 2i)$
23. $-i(4 - 8i)$
24. $(5 - 7i)(-4 - 3i)$
25. $(-2 + 5i)(-1 + 4i)$
26. $(-1 - 5i)(-1 + 5i)$
27. $(8 - 3i)(8 + 3i)$
28. $\frac{7i}{8 + i}$
29. $\frac{6i}{3 - i}$
30. $\frac{-2 - 5i}{3i}$
31. $\frac{4 + 9i}{12i}$
32. $\frac{7 + 4i}{2 - 3i}$
33. $\frac{-1 - 6i}{5 + 9i}$