4.7 – Complete the Square

Example 1 Solve $x^2 - 8x + 16 = 25$ by finding square roots

$(x-4)(x-4) = 25$

$\sqrt{(x-4)^2} = \sqrt{25}$

$x - 4 = \pm 5$

$+4 \quad +4$

$x = 4 \pm 5$

$x = 4 + 5 = 9$

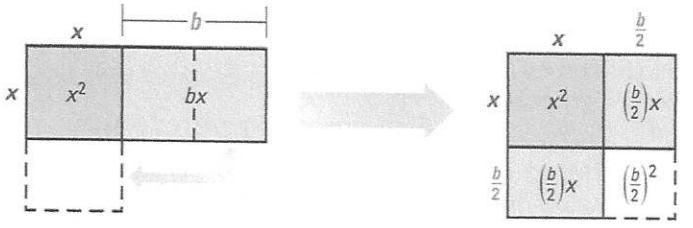
$x = 4 - 5 = -1$

KEY CONCEPT *For Your Notebook*

Completing the Square

Words To complete the square for the expression $x^2 + bx$, add $\left(\frac{b}{2}\right)^2$.

Diagrams In each diagram, the combined area of the shaded regions is $x^2 + bx$. Adding $\left(\frac{b}{2}\right)^2$ completes the square in the second diagram.



Algebra $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)\left(x + \frac{b}{2}\right) = \left(x + \frac{b}{2}\right)^2$

Example 2 Find the value of c that makes $x^2 + 16x + c$ a perfect square trinomial, then write the expression as the square of a binomial

$\left(\frac{b}{2}\right)^2 \quad b = 16$

$\left(\frac{16}{2}\right)^2 = 8^2$

$= 64$

$x^2 + 16x + 64$

$(x+8)(x+8)$

$= (x+8)^2$

Example 3 Solve by completing the square

$x^2 - 12x + 4 = 0$

$-4 \quad -4$

$\left(\frac{b}{2}\right)^2 = \left(\frac{-12}{2}\right)^2 = (-6)^2 = 36$

$x^2 - 12x = -4 + 36$

$x^2 - 12x + 36 = 32$

$\sqrt{(x-6)^2} = \sqrt{32}$

$x - 6 = \sqrt{16}\sqrt{2}$

$-6 \quad -6$

$x = -6 \pm 4\sqrt{2}$

Example 4 Solve by completing the square

$$\frac{2x^2 + 8x + 14 = 0}{2}$$

$$x^2 + 4x + 7 = 0 \qquad \left(\frac{b}{2}\right)^2 = \left(\frac{4}{2}\right)^2 = 2^2 = 4$$

$$-7 \quad -7$$

$$x^2 + 4x + 4 = -7 + 4$$

$$\sqrt{(x+2)^2} = \sqrt{-3}$$

$$x + 2 = \pm i\sqrt{3} \rightarrow x = -2 \pm i\sqrt{3}$$

$$-2 \quad -2$$

HW: (4-10 even) (13-21 all) (23-33 odd)

EXAMPLE 1 on p. 284 for Exs. 3–12

SOLVING BY SQUARE ROOTS **Solve the equation by finding square roots.**

3. $x^2 + 4x + 4 = 9$
4. $x^2 + 10x + 25 = 64$
5. $n^2 + 16n + 64 = 36$
6. $m^2 - 2m + 1 = 144$
7. $x^2 - 22x + 121 = 13$
8. $x^2 - 18x + 81 = 5$
9. $t^2 + 8t + 16 = 45$
10. $4u^2 + 4u + 1 = 75$
11. $9x^2 - 12x + 4 = -3$

12. ★ **MULTIPLE CHOICE** What are the solutions of $x^2 - 4x + 4 = -1$?

Ⓐ $2 \pm i$ Ⓑ $-2 \pm i$ Ⓒ $-3, -1$ Ⓓ $1, 3$

EXAMPLE 2 on p. 285 for Exs. 13–21

FINDING C **Find the value of c that makes the expression a perfect square trinomial. Then write the expression as the square of a binomial.**

13. $x^2 + 6x + c$
14. $x^2 + 12x + c$
15. $x^2 - 24x + c$
16. $x^2 - 30x + c$
17. $x^2 - 2x + c$
18. $x^2 + 50x + c$
19. $x^2 + 7x + c$
20. $x^2 - 13x + c$
21. $x^2 - x + c$

EXAMPLES 3 and 4 on pp. 285–286 for Exs. 22–34

COMPLETING THE SQUARE **Solve the equation by completing the square.**

22. $x^2 + 4x = 10$
23. $x^2 + 8x = -1$
24. $x^2 + 6x - 3 = 0$
25. $x^2 + 12x + 18 = 0$
26. $x^2 - 18x + 86 = 0$
27. $x^2 - 2x + 25 = 0$
28. $2k^2 + 16k = -12$
29. $3x^2 + 42x = -24$
30. $4x^2 - 40x - 12 = 0$
31. $3s^2 + 6s + 9 = 0$
32. $7t^2 + 28t + 56 = 0$
33. $6r^2 + 6r + 12 = 0$

34. ★ **MULTIPLE CHOICE** What are the solutions of $x^2 + 10x + 8 = -5$?

Ⓐ $5 \pm 2\sqrt{3}$ Ⓑ $5 \pm 4\sqrt{3}$ Ⓒ $-5 \pm 2\sqrt{3}$ Ⓓ $-5 \pm 4\sqrt{3}$