

5.2a – Evaluate Polynomial Functions

Polynomial Functions – the sums and differences of monomial functions

Degree of a Polynomial – the term with the highest variable exponent

Leading Coefficient – the coefficient of the variable with the highest exponent

Degree	Type	Standard Form	Example
0	Constant	$f(x) = a_0$	$f(x) = -14 \cancel{x^0} = 1$
1	Linear	$f(x) = a_1x + a_0$	$f(x) = 5x - 7$
2	Quadratic	$f(x) = a_2x^2 + a_1x + a_0$	$f(x) = 2x^2 + x - 9$
3	Cubic	$f(x) = a_3x^3 + a_2x^2 + a_1x + a_0$	$f(x) = x^3 - x^2 + 3x$
4	Quartic	$f(x) = a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$	$f(x) = x^4 + 2x - 1$

Example 1 Decide whether the function is a polynomial function. If so, write it in standard form and state its degree, type, and leading coefficient

a. $h(x) = x^4 - \frac{1}{4}x^2 + 3$

Yes, $x^4 - \frac{1}{4}x^2 + 3$

D \Rightarrow 4, Quartic

-LC \Rightarrow 1

b. $g(x) = 7x - \sqrt{3} + \pi x^2$

Yes, $\pi x^2 + 7x - \sqrt{3}$

D \Rightarrow 2, Quadratic

LC $\Rightarrow \pi$

c. $f(x) = 5x^2 + 3x^{-1} - x$

No. -1 is not a whole # exponent

d. $k(x) = x + 2^x - 0.6x^5$

No. A polynomial cannot have a variable exponent

Example 2 Use direct substitution to evaluate when $x = 3$

$f(x) = 2x^4 - 5x^3 - 4x + 8$

$$\begin{aligned} f(3) &= 2(3)^4 - 5(3)^3 - 4(3) + 8 \\ &= 2(81) - 5(27) - 12 + 8 \\ &= 162 - 135 - 4 \end{aligned}$$

$f(3) = 23$

Example 3 Use synthetic substitution to evaluate $f(x) = 2x^4 - 5x^3 - 4x + 8$ when $x = 3$

$$\begin{array}{r} 3 | 2 \quad -5 \quad 0 \quad -4 \quad 8 \\ \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\ 2 \quad 1 \quad 3 \quad 5 \quad | 23 \end{array} \rightarrow f(3) = 23$$

HW: (3-12), (15-20)

EXAMPLE 1 3-8	POLYNOMIAL FUNCTIONS Decide whether the function is a polynomial function. If so, write it in standard form and state its degree, type, and leading coefficient.
	3. $f(x) = 8 - x^2$ 4. $f(x) = 6x + 8x^4 - 3$ 5. $g(x) = \pi x^4 + \sqrt{6}$
	6. $h(x) = x^3\sqrt{10} + 5x^{-2} + 1$ 7. $h(x) = -\frac{5}{2}x^3 + 3x - 10$ 8. $g(x) = 8x^3 - 4x^2 + \frac{2}{x}$
EXAMPLE 2 3-14	DIRECT SUBSTITUTION Use direct substitution to evaluate the polynomial function for the given value of x .
	9. $f(x) = 5x^3 - 2x^2 + 10x - 15; x = -1$ 10. $f(x) = 8x + 5x^4 - 3x^2 - x^3; x = 2$ 11. $g(x) = 4x^3 - 2x^5; x = -3$ 12. $h(x) = 6x^3 - 25x + 20; x = 5$ 13. $h(x) = x + \frac{1}{2}x^4 - \frac{3}{4}x^3 + 10; x = -4$ 14. $g(x) = 4x^5 + 6x^3 + x^2 - 10x + 5; x = -2$
EXAMPLE 3 15-23	SYNTHETIC SUBSTITUTION Use synthetic substitution to evaluate the polynomial function for the given value of x .
	15. $f(x) = 5x^3 - 2x^2 - 8x + 16; x = 3$ 16. $f(x) = 8x^4 + 12x^3 + 6x^2 - 5x + 9; x = -2$ 17. $g(x) = x^3 + 8x^2 - 7x + 35; x = -6$ 18. $h(x) = -8x^3 + 14x - 35; x = 4$ 19. $f(x) = -2x^4 + 3x^3 - 8x + 13; x = 2$ 20. $g(x) = 6x^5 + 10x^3 - 27; x = -3$ 21. $h(x) = -7x^3 + 11x^2 + 4x; x = 3$ 22. $f(x) = x^4 + 3x - 20; x = 4$