5.2b – Graph Polynomial Functions

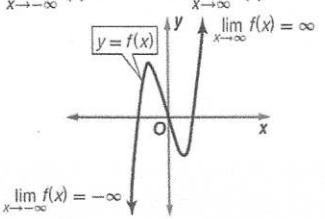
End Behavior of Polynomial Functions

Lead Term Test

The end behavior of any non-constant polynomial function $f(x) = a_n x^n + \cdots + a_1 x + a_0$ can be described in one of the following four ways, as determined by the degree n of the polynomial and its leading coefficient a_n.

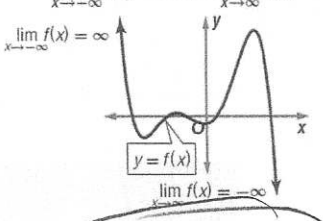
n odd, a_n positive

$\lim_{x \to -\infty} f(x) = -\infty$ and $\lim_{x \to \infty} f(x) = \infty$



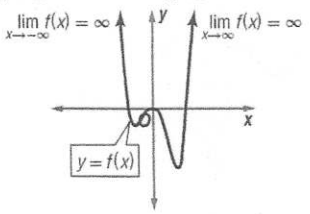
n odd, a_n negative

$\lim_{x \to -\infty} f(x) = \infty$ and $\lim_{x \to \infty} f(x) = -\infty$



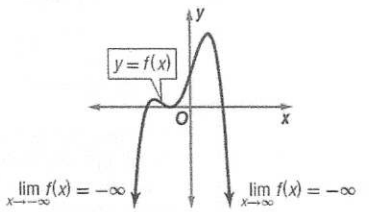
n even, a_n positive

$\lim_{x \to -\infty} f(x) = \infty$ and $\lim_{x \to \infty} f(x) = \infty$



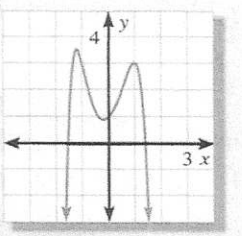
n even, a_n negative

$\lim_{x \to -\infty} f(x) = -\infty$ and $\lim_{x \to \infty} f(x) = -\infty$



Example 4

What is true about the degree and leading coefficient of the polynomial function whose graph is shown?



(A) Degree is odd; leading coefficient is positive

(B) Degree is odd; leading coefficient is negative

(C) Degree is even; leading coefficient is positive

(D) Degree is even; leading coefficient is negative

D

Graph Polynomial Functions

Step 1 Use a table of values to plot points to determine the shape of the graphs middle portion

Step 2 Then use what you know about end behavior to sketch the ends of the graph

Example 5 Graph

a. $f(x) = -x^3 + x^2 + 3x - 3$

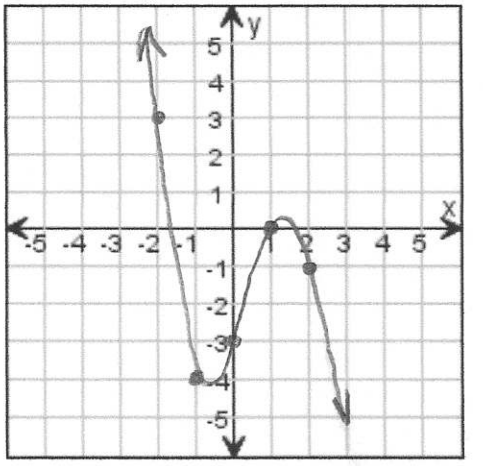
x	-3	-2	-1	0	1	2	3
y	24	3	-4	-3	0	-1	-12

$f(-3) = -(-3)^3 + (-3)^2 + 3(-3) - 3$

$= 27 + 9 - 9 - 3$

$f(-2) = 3$ $f(-1) = -4$ $f(0) = -3$

$f(1) = 0$ $f(2) = -1$ $f(3) = -12$



b. $f(x) = x^4 - x^3 - 4x^2 + 4$

x	-3	-2	-1	0	1	2	3
y	76	12	2	4	0	-4	22

$f(-3) = (-3)^4 - (-3)^3 - 4(-3)^2 + 4$

$= 81 + 27 - 36 + 4$

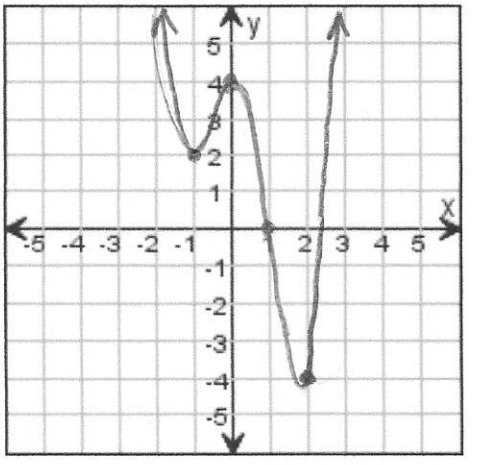
$= 76$

$f(-2) = 16 + 8 - 16 + 4$

$= 12$

$f(-1) = 2$ $f(0) = 4$ $f(1) = 0$

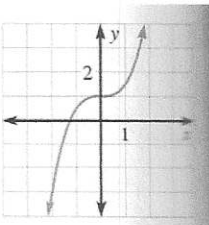
$f(2) = -4$ $f(3) = 22$



HW: (24), (28-33), (46, 48)

EXAMPLE 4 on p. 339 for Exs. 24–27

24. ★ **MULTIPLE CHOICE** The graph of a polynomial function is shown. What is true about the function's degree and leading coefficient?



Ⓐ The degree is odd and the leading coefficient is positive.

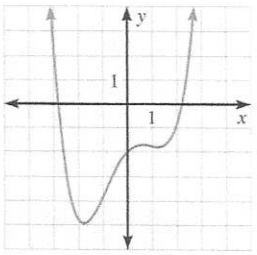
Ⓑ The degree is odd and the leading coefficient is negative.

Ⓒ The degree is even and the leading coefficient is positive.

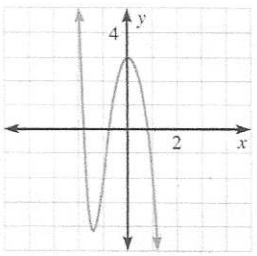
Ⓓ The degree is even and the leading coefficient is negative.

USING END BEHAVIOR ***Describe*** **the degree and leading coefficient of the polynomial function whose graph is shown.**

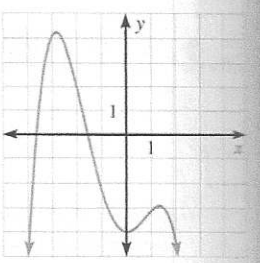
25.



26.



27.



DESCRIBING END BEHAVIOR ***Describe*** **the end behavior of the graph of the polynomial function by completing these statements:** $f(x) \to \underline{?}$ **as** $x \to -\infty$ **and** $f(x) \to \underline{?}$ **as** $x \to +\infty$**.**

28. $f(x) = 10x^4$

29. $f(x) = -x^6 + 4x^3 - 3x$

30. $f(x) = -2x^3 + 7x - 4$

31. $f(x) = x^7 + 3x^4 - x^2$

32. $f(x) = 3x^{10} - 16x$

33. $f(x) = -6x^5 + 14x^2 + 20$

34. $f(x) = 0.2x^3 - x + 45$

35. $f(x) = 5x^8 + 8x^7$

36. $f(x) = -x^{273} + 500x^{271}$

37. ★ **OPEN-ENDED MATH** Write a polynomial function f of degree 5 such that the end behavior of the graph of f is given by $f(x) \to +\infty$ as $x \to -\infty$ and $f(x) \to -\infty$ as $x \to +\infty$. Then graph the function to verify your answer.

EXAMPLE 5 on p. 340 for Exs. 38–50

GRAPHING POLYNOMIALS **Graph the polynomial function.**

38. $f(x) = x^3$

39. $f(x) = -x^4$

40. $f(x) = x^5 + 3$

41. $f(x) = x^4 - 2$

42. $f(x) = -x^3 + 5$

43. $f(x) = x^3 - 5x$

44. $f(x) = -x^4 + 8x$

45. $f(x) = x^5 + x$

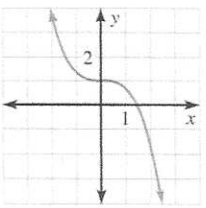
46. $f(x) = -x^3 + 3x^2 - 2x + 5$

47. $f(x) = x^5 + x^2 - 4$

48. $f(x) = x^4 - 5x^2 + 6$

49. $f(x) = -x^4 + 3x^3 - x + 1$

50. ★ **MULTIPLE CHOICE** Which function is represented by the graph shown?



Ⓐ $f(x) = \frac{1}{3}x^3 + 1$ Ⓑ $f(x) = -\frac{1}{3}x^3 + 1$

Ⓒ $f(x) = \frac{1}{3}x^3 - 1$ Ⓓ $f(x) = -\frac{1}{3}x^3 - 1$