5.3 – Add, Subtract, and Multiply Polynomials

To add and subtract polynomials, add or subtract the coefficients of like terms

Example 1 Add polynomials vertically and horizontally

a. $2x^3 - 5x^2 + 3x - 9$ add $x^3 + 6x^2 + 11$ vertically

$$\begin{array}{rl} & 2x^3 - 5x^2 + 3x - 9 \\ + & x^3 + 6x^2 \qquad\quad + 11 \\ \hline & 3x^3 + x^2 + 3x + 2 \end{array}$$

b. $3y^3 - 2y^2 - 7y$ add $-4y^2 + 2y - 5$ horizontally

$(3y^3 - 2y^2 - 7y) + (-4y^2 + 2y - 5) = 3y^3 - 6y^2 - 5y - 5$

Example 2 Subtract polynomials vertically and horizontally

a. Subtract $3x^3 + 2x^2 - x + 7$ from $8x^3 - x^2 - 5x + 1$ vertically

$$\begin{array}{rl} & 8x^3 - x^2 - 5x + 1 \\ - & (3x^3 + 2x^2 - x + 7) \\ \hline & 5x^3 - 3x^2 - 4x - 6 \end{array}$$

b. Subtract $5z^2 - \overset{z}{\cancel{s}} + 3$ from $4z^2 + 9z - 12$ horizontally

$(4z^2 + 9z - 12) - (5z^2 - z + 3) = -z^2 + 10z - 15$

Example 3 Multiply polynomials vertically and horizontally

a. Multiply $-2y^2 + 3y - 6$ by $y - 2$ vertically

$$\begin{array}{rl} & -2y^2 + 3y - 6 \\ \times & \qquad\qquad y - 2 \\ \hline & 4y^2 - 6y + 12 \\ & -2y^2 + 3y^2 - 6y \end{array}$$

$= -2y^3 + 7y^2 - 12y + 12$

b. Multiply $x + 3$ by $3x^2 - 2x + 4$ horizontally

$(x+3)(3x^2-2x+4)$

$= 3x^3 - 2x^2 + 4x + 9x^2 - 6x + 12$

$= 3x^3 + 7x^2 - 2x + 12$

Example 4 Multiply three binomials

Multiply $x - 5$, $x + 1$, and $x + 3$ horizontally

$(x-5)(x+1)(x+3)$

$= (x-5)(x^2 + 4x + 3)$

$= x^3 + 4x^2 + 3x - 5x^2 - 20x - 15$

$= x^3 - x^2 - 17x - 15$

Example 5 Using Special Product Patterns

a. $(3t + 4)(3t - 4)$

$= 9t^2 - 16$

b. $(8x - 3)^2$

$= 64x^2 - 48x + 9$

c. $(pq + 5)^3$

$= p^3q^3 + 15p^2q^2 + 75pq + 125$

KEY CONCEPT *For Your Notebook*

Special Product Patterns

Sum and Difference	Example
$(a + b)(a - b) = a^2 - b^2$	$(x + 4)(x - 4) = x^2 - 16$
Square of a Binomial	**Example**
$(a + b)^2 = a^2 + 2ab + b^2$	$(y + 3)^2 = y^2 + 6y + 9$
$(a - b)^2 = a^2 - 2ab + b^2$	$(3z^2 - 5)^2 = 9z^4 - 30z^2 + 25$
Cube of a Binomial	**Example**
$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$	$(x + 2)^3 = x^3 + 6x^2 + 12x + 8$
$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$	$(p - 3)^3 = p^3 - 9p^2 + 27p - 27$

HW: (4-10 even), (17-23 odd), (28,29), (38-42)

EXAMPLES 1 and 2
on p. 346
for Exs. 3–15

ADDING AND SUBTRACTING POLYNOMIALS **Find the sum or difference.**

3. $(3x^2 - 5) + (7x^2 - 3)$
4. $(x^2 - 3x + 5) - (-4x^2 + 8x + 9)$
5. $(4y^2 + 9y - 5) - (4y^2 - 5y + 3)$
6. $(z^2 + 5z - 7) + (5z^2 - 11z - 6)$
7. $(3s^3 + s) + (4s^3 - 2s^2 + 7s + 10)$
8. $(2a^2 - 8) - (a^3 + 4a^2 - 12a + 4)$
9. $(5c^2 + 7c + 1) + (2c^3 - 6c + 8)$
10. $(4t^3 - 11t^2 + 4t) - (-7t^2 - 5t + 8)$
11. $(5b - 6b^3 + 2b^4) - (9b^3 + 4b^4 - 7)$
12. $(3y^2 - 6y^4 + 5 - 6y) + (5y^4 - 6y^3 + 4y)$
13. $(x^4 - x^3 + x^2 - x + 1) + (x + x^4 - 1 - x^2)$
14. $(8v^4 - 2v^2 + v - 4) - (3v^3 - 12v^2 + 8v)$

15. ★ **MULTIPLE CHOICE** What is the result when $2x^4 - 8x^2 - x + 10$ is subtracted from $8x^4 - 4x^3 - x + 2$?

Ⓐ $-6x^4 + 4x^3 - 8x^2 + 8$
Ⓑ $6x^4 - 4x^3 + 8x^2 - 8$
Ⓒ $10x^4 - 8x^3 - 4x^2 + 12$
Ⓓ $6x^4 + 4x^3 - 2x - 8$

EXAMPLE 3
on p. 347
for Exs. 16–25

MULTIPLYING POLYNOMIALS **Find the product of the polynomials.**

16. $x(2x^2 - 5x + 7)$
17. $5x^2(6x + 2)$
18. $(y - 7)(y + 6)$
19. $(3z + 1)(z - 3)$
20. $(w + 4)(w^2 + 6w - 11)$
21. $(2a - 3)(a^2 - 10a - 2)$
22. $(5c^2 - 4)(2c^2 + c - 3)$
23. $(-x^2 + 4x + 1)(x^2 - 8x + 3)$
24. $(-d^2 + 4d + 3)(3d^2 - 7d + 6)$
25. $(3y^2 + 6y - 1)(4y^2 - 11y - 5)$

ERROR ANALYSIS ***Describe*** **and correct the error in simplifying the expression.**

26.
$$(x^2 - 3x + 4) - (x^3 + 7x - 2)$$
$$= x^2 - 3x + 4 - x^3 + 7x - 2$$
$$= -x^3 + x^2 + 4x + 2$$

27.
$$(2x - 7)^3 = (2x)^3 - 7^3$$
$$= 8x^3 - 343$$

EXAMPLE 4
on p. 347
for Exs. 28–37

MULTIPLYING THREE BINOMIALS **Find the product of the binomials.**

28. $(x + 4)(x - 6)(x - 5)$
29. $(x + 1)(x - 7)(x + 3)$
30. $(z - 4)(-z + 2)(z + 8)$
31. $(a - 6)(2a + 5)(a + 1)$
32. $(3p + 1)(p + 3)(p + 1)$
33. $(b - 2)(2b - 1)(-b + 1)$
34. $(2s + 1)(3s - 2)(4s - 3)$
35. $(w - 6)(4w - 1)(-3w + 5)$
36. $(4x - 1)(-2x - 7)(-5x - 4)$
37. $(3q - 8)(-9q + 2)(q - 2)$

EXAMPLE 5
on p. 348
for Exs. 38–47

SPECIAL PRODUCTS **Find the product.**

38. $(x + 5)(x - 5)$
39. $(w - 9)^2$
40. $(y + 4)^3$
41. $(2c + 5)^2$
42. $(3t - 4)^3$
43. $(5p - 3)(5p + 3)$
44. $(7x - y)^3$
45. $(2a + 9b)(2a - 9b)$
46. $(3z + 7y)^3$