

## 5.4 – Factor Polynomial Equations

A Polynomial is completely factored if it is written as a product of unfactorable polynomials with integer coefficients.

Example 1 Find a common monomial factor

a.  $x^3 + 2x^2 - 15x$

$$x(x^2 + 2x - 15)$$

$$x(x+5)(x-3)$$

c.  $4z^4 - 16z^3 + 16z^2$

$$4z^2(z^2 - 4z + 4)$$

$$4z^2(z-2)^2$$

b.  $2y^5 - 18y^3$

$$2y^3(y^2 - 9)$$

$$2y^3(y+3)(y-3)$$

Factoring Special Patterns

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

Example 2 Factor the Sum and Difference of Cubes

a.  $x^3 + 64 \quad a=x \quad b=4$

$$(x+4)(x^2 - 4x + 16)$$

b.  $16z^5 - 250z^2 \quad a=2z \quad b=5$

$$2z^2(8z^3 - 125)$$

$$2z^2(2z-5)(4z^2 + 10z + 25)$$

Example 3 Factor by grouping

a.  $x^3 - 3x^2 - 16x + 48$

$$(x^3 - 3x^2)(-16x + 48)$$

$$x^2(x-3) - 16(x-3)$$

$$(x-3)(x^2 - 16)$$

$$(x-3)(x+4)(x-4)$$

**Example 4** Factor Polynomials in quadratic form

a.  $16x^4 - 81$

$$(4x^2)^2 - 81$$

$$(4x^2 + 9)(4x^2 - 9)$$

$$(4x^2 + 9)(2x + 3)(2x - 3)$$

b.  $2p^8 + 10p^5 + 12p^2$

$$2p^2(p^6 + 5p^3 + 6)$$

$$2p^2(u^2 + 5u + 6)$$

$$2p^2(u+2)(u+3)$$

$$2p^2(p^3 + 2)(p^3 + 3)$$

let  $u = p^3$

HW: (4-28 even)

**EXAMPLE 1**

on p. 353  
for Exs. 3-9

**MONOMIAL FACTORS** Factor the polynomial completely.

3.  $14x^2 - 21x$

4.  $30b^3 - 54b^2$

5.  $c^3 + 9c^2 + 18c$

6.  $z^3 - 6z^2 - 72z$

7.  $3y^5 - 48y^3$

8.  $54m^5 + 18m^4 + 9m^3$

9. ★ **MULTIPLE CHOICE** What is the complete factorization of  $2x^7 - 32x^3$ ?

(A)  $2x^3(x + 2)(x - 2)(x^2 + 4)$

(B)  $2x^3(x^2 + 2)(x^2 - 2)$

(C)  $2x^3(x^2 + 4)^2$

(D)  $2x^3(x + 2)^2(x - 2)^2$

**EXAMPLE 2**

p. 354  
Exs. 10-17

**SUM OR DIFFERENCE OF CUBES** Factor the polynomial completely.

10.  $x^3 + 8$

11.  $y^3 - 64$

12.  $27m^3 + 1$

13.  $125n^3 + 216$

14.  $27a^3 - 1000$

15.  $8c^3 + 343$

16.  $192w^3 - 3$

17.  $-5z^3 + 320$

**EXAMPLE 3**

p. 354  
Exs. 18-23

**FACTORING BY GROUPING** Factor the polynomial completely.

18.  $x^3 + x^2 + x + 1$

19.  $y^3 - 7y^2 + 4y - 28$

20.  $n^3 + 5n^2 - 9n - 45$

21.  $3m^3 - m^2 + 9m - 3$

22.  $25s^3 - 100s^2 - s + 4$

23.  $4c^3 + 8c^2 - 9c - 18$

**EXAMPLE 4**

p. 355  
Exs. 24-29

**QUADRATIC FORM** Factor the polynomial completely.

24.  $x^4 - 25$

25.  $a^4 + 7a^2 + 6$

26.  $3s^4 - s^2 - 24$

27.  $32z^5 - 2z$

28.  $36m^6 + 12m^4 + m^2$

29.  $15x^5 - 72x^3 - 108x$