5.5 – Apply the Remainder and Factor Theorems

Polynomial Long Division

Example 1 Divide $3x^4 - 5x^3 + 4x - 6$ *by* $x^2 - 3x + 5$

$$
\begin{array}{r}
3x^2 + 4x - 3 \\
x^2 - 3x + 5 \overline{\big) 3x^4 - 5x^3 + 0x^2 + 4x - 6} \\
-(3x^4 - 9x^3 + 15x^2) \\
\hline
4x^3 - 15x^2 + 4x \\
-(4x^3 - 12x^2 + 20x) \\
\hline
-3x^2 - 16x - 6 \\
-(-3x^2 + 9x - 15) \\
\hline
-25x + 9 \leftarrow \text{remainder}
\end{array}
$$

① $\frac{3x^4}{x^2} = 3x^2$

② $3x^2(x^2 - 3x + 5)$

③ subtract

④ $\frac{4x^3}{x^2} = 4x$

⑤ $4x(x^2 - 3x + 5)$

⑥ $\frac{-3x^2}{x^2} = -3$

⑦ $-3(x^2 - 3x + 5)$

$$\boxed{3x^2 + 4x - 3 + \frac{-25x + 9}{x^2 - 3x + 5}}$$

Example 2 Divide $x^3 + 5x^2 - 7x + 2$ *by* $x - 2$

$$
\begin{array}{r}
x^2 + 7x + 7 \\
x - 2 \overline{\big) x^3 + 5x^2 - 7x + 2} \\
-(x^3 - 2x^2) \\
\hline
7x^2 - 7x \\
-(7x^2 - 14x) \\
\hline
7x + 2 \\
-(7x - 14) \\
\hline
16 \quad \text{remainder}
\end{array}
$$

$$\boxed{x^2 + 7x + 7 + \frac{16}{x - 2}}$$

Use Synthetic Division

$x - 2 = 0$
$+2 \quad +2$
$x = 2$

$$
\begin{array}{r|rrrr}
2 & 1 & 5 & -7 & 2 \\
 & & 2 & 14 & 14 \\
\hline
 & 1 & 7 & 7 & r = 16
\end{array}
$$

$$x^2 + 7x + 7 + \frac{16}{x - 2}$$

Remainder Theorem–If a polynomial $f(x)$ is divided by $x - k$, then the remainder is $r = f(k)$

Example 3 Use the remainder theorem. Divide $2x^3 + x^2 - 8x + 5$ *by* $x + 3$

$x + 3 = 0$
$-3 \quad -3$
$x = -3$

$$\begin{array}{r|rrrr} -3 & 2 & 1 & -8 & 5 \\ & & -6 & 15 & -21 \\ \hline & 2 & -5 & 7 & \boxed{r = -16} \end{array}$$

$$\boxed{2x^2 - 5x + 7 + \frac{-16}{x+3}}$$

Factor Theorem-A polynomial $f(x)$ has a factor $x - k$ if and only if $f(k) = 0$

Example 4 Factor $3x^3 - 4x^2 - 28x - 16$ completely given that $x + 2$ is a factor.

$x + 2 = 0$
$-2 \quad -2$
$x = -2$

$$\begin{array}{r|rrrr} -2 & 3 & -4 & -28 & -16 \\ & & -6 & 20 & 16 \\ \hline & 3 & -10 & -8 & \boxed{0} \end{array}$$

$(3x^2 - 10x - 8)(x + 2)$

$x^2 - 10x - 24$

$(x + \frac{2}{3})(x - \frac{12}{3})$

$\boxed{(3x+2)(x-4)(x+2)}$

HW: (3, 9, 11, 17, 21, 22)

EXAMPLES 1 and 2
on pp. 362–363
for Exs. 3–10

USING LONG DIVISION **Divide using polynomial long division.**

3. $(x^2 + x - 17) \div (x - 4)$
4. $(3x^2 - 11x - 26) \div (x - 5)$
5. $(x^3 + 3x^2 + 3x + 2) \div (x - 1)$
6. $(8x^2 + 34x - 1) \div (4x - 1)$
7. $(3x^3 + 11x^2 + 4x + 1) \div (x^2 + x)$
8. $(7x^3 + 11x^2 + 7x + 5) \div (x^2 + 1)$
9. $(5x^4 - 2x^3 - 7x^2 - 39) \div (x^2 + 2x - 4)$
10. $(4x^4 + 5x - 4) \div (x^2 - 3x - 2)$

EXAMPLE 3
on p. 363
for Exs. 11–20

USING SYNTHETIC DIVISION **Divide using synthetic division.**

11. $(2x^2 - 7x + 10) \div (x - 5)$
12. $(4x^2 - 13x - 5) \div (x - 2)$
13. $(x^2 + 8x + 1) \div (x + 4)$
14. $(x^2 + 9) \div (x - 3)$
15. $(x^3 - 5x^2 - 2) \div (x - 4)$
16. $(x^3 - 4x + 6) \div (x + 3)$
17. $(x^4 - 5x^3 - 8x^2 + 13x - 12) \div (x - 6)$
18. $(x^4 + 4x^3 + 16x - 35) \div (x + 5)$

ERROR ANALYSIS ***Describe*** **and correct the error in using synthetic division to divide $x^3 - 5x + 3$ by $x - 2$.**

19.

2	1	0	−5	3
		2	4	−2
	1	2	−1	1

$$\frac{x^3 - 5x + 3}{x - 2} = x^3 + 2x^2 - x + 1$$

20.

2	1	−5	3
		2	−6
	1	−3	−3

$$\frac{x^3 - 5x + 3}{x - 2} = x^2 - 3x - \frac{3}{x - 2}$$

EXAMPLE 4
on p. 364
for Exs. 21–28

FACTOR **Given polynomial $f(x)$ and a factor of $f(x)$, factor $f(x)$ completely.**

21. $f(x) = x^3 - 10x^2 + 19x + 30;\ x - 6$
22. $f(x) = x^3 + 6x^2 + 5x - 12;\ x + 4$
23. $f(x) = x^3 - 2x^2 - 40x - 64;\ x - 8$
24. $f(x) = x^3 + 18x^2 + 95x + 150;\ x + 10$
25. $f(x) = x^3 + 2x^2 - 51x + 108;\ x + 9$
26. $f(x) = x^3 - 9x^2 + 8x + 60;\ x + 2$
27. $f(x) = 2x^3 - 15x^2 + 34x - 21;\ x - 1$
28. $f(x) = 3x^3 - 2x^2 - 61x - 20;\ x - 5$