6.1 – Evaluate *n*th Roots and use Rational Exponents

If $2^3 = 8$, we say 2 is the cube root of 8. Similarly, if $b^n = a$, we can also say that b is an nth root of a.

An nth root of a is written as: $\sqrt[n]{a}$, where n is the INDEX of the radical.

Example 1 Find nth Roots

a. $n = 3, a = -216$

$-216^{1/3} = \sqrt[3]{-216} = -6$

b. $n = 4, a = 81$

$81^{1/4} = \sqrt[4]{81} = \pm 3$

Rational Exponents

$a^{m/n} = \left(a^{1/n}\right)^m = \left(\sqrt[n]{a}\right)^m$

$a^{-m/n} = \frac{1}{a^{m/n}} = \frac{1}{\left(a^{1/n}\right)^m} = \frac{1}{\left(\sqrt[n]{a}\right)^m}, a \neq 0$

Example 2 Evaluate expression with rational exponents

a. $16^{3/2}$

$\left(\sqrt{16}\right)^3 = 4^3 = 64$

b. $32^{-3/5}$

$\left(\sqrt[5]{32}\right)^{-3} = 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

Example 3 Estimate nth roots with a calculator

a. $9^{1/5} \approx 1.552$

b. $12^{3/8} \approx 2.539$

c. $\left(\sqrt[4]{7}\right)^3$

$7^{3/4} \approx 4.304$

Example 4 Solving equations with nth roots

a. $4x^5 = 128$

$\frac{4x^5}{4} = \frac{128}{4}$

$x^5 = 32$

$\sqrt[5]{x^5} = \sqrt[5]{32}$

$x = 2$

b. $(x-3)^4 = 21$

$\sqrt[4]{(x-3)^4} = \sqrt[4]{21}$

$x - 3 = \pm\sqrt[4]{21}$

$+3 \qquad +3$

$x = 3 \pm \sqrt[4]{21}$

HW: (8-28), (34-40 even), (48-54 even)

EXAMPLE 1
on p. 414
for Exs. 3–20

MATCHING EXPRESSIONS **Match the expression in rational exponent notation with the equivalent expression in radical notation.**

3. $2^{1/3}$ 4. $2^{3/2}$ 5. $2^{2/3}$ 6. $2^{1/2}$

A. $(\sqrt{2})^3$ B. $\sqrt{2}$ C. $\sqrt[3]{2}$ D. $(\sqrt[3]{2})^2$

USING RATIONAL EXPONENT NOTATION **Rewrite the expression using rational exponent notation.**

7. $\sqrt[3]{12}$ 8. $\sqrt[5]{8}$ 9. $(\sqrt[3]{10})^7$ 10. $(\sqrt[8]{15})^3$

USING RADICAL NOTATION **Rewrite the expression using radical notation.**

11. $5^{1/4}$ 12. $7^{1/3}$ 13. $14^{2/5}$ 14. $21^{9/4}$

FINDING NTH ROOTS **Find the indicated real *n*th root(s) of *a*.**

15. $n = 2, a = 64$ 16. $n = 3, a = -27$ 17. $n = 4, a = 0$

18. $n = 3, a = 343$ 19. $n = 4, a = -16$ 20. $n = 5, a = -32$

EXAMPLE 2
on p. 415
for Exs. 21–33

EVALUATING EXPRESSIONS **Evaluate the expression without using a calculator.**

21. $\sqrt[6]{64}$ 22. $8^{1/3}$ 23. $16^{3/2}$ 24. $\sqrt[3]{-125}$

25. $27^{2/3}$ 26. $(-243)^{1/5}$ 27. $(\sqrt[3]{8})^{-2}$ 28. $(\sqrt[3]{-64})^4$

29. $(\sqrt[4]{16})^{-7}$ 30. $25^{3/2}$ 31. $64^{-2/3}$ 32. $\dfrac{1}{81^{-3/4}}$

33. ★ **MULTIPLE CHOICE** What is the value of $128^{5/7}$?

(A) 8 (B) 16 (C) 32 (D) 64

EXAMPLE 3
on p. 415
for Exs. 34–46

APPROXIMATING ROOTS **Evaluate the expression using a calculator. Round the result to two decimals places when appropriate.**

34. $\sqrt[5]{32{,}768}$ 35. $\sqrt[7]{1695}$ 36. $\sqrt[9]{-230}$ 37. $85^{1/6}$

38. $25^{-1/3}$ 39. $20{,}736^{1/4}$ 40. $(\sqrt[4]{187})^3$ 41. $(\sqrt{6})^{-5}$

42. $(\sqrt[5]{-8})^8$ 43. $86^{-5/6}$ 44. $1974^{2/7}$ 45. $\dfrac{1}{(-17)^{3/5}}$

EXAMPLE 4
on p. 416
for Exs. 48–58

ERROR ANALYSIS ***Describe*** **and correct the error in solving the equation.**

48.
$x^3 = 27$
$x = \sqrt[3]{27}$
$x = 9$

49.
$x^4 = 81$
$x = \sqrt[4]{81}$
$x = 3$

SOLVING EQUATIONS **Solve the equation. Round the result to two decimal places when appropriate.**

50. $x^3 = 125$ 51. $5x^3 = 1080$ 52. $x^6 + 36 = 100$

53. $(x - 5)^4 = 256$ 54. $x^5 = -48$ 55. $7x^4 = 56$

56. $x^3 + 40 = 25$ 57. $(x + 10)^5 = 70$ 58. $x^6 - 34 = 181$