6.2 – Apply Properties of Rational Exponents

The same properties that apply to whole number exponents also apply to rational exponents. Here they are again:

Property	Example
1. $a^m \cdot a^n = a^{m+n}$	$5^{1/2} \cdot 5^{3/2} = 5^{(1/2+3/2)} = 5^2 = 25$
2. $(a^m)^n = a^{mn}$	$(3^{5/2})^2 = 3^{(5/2 \cdot 2)} = 3^5 = 243$
3. $(ab)^m = a^m b^m$	$(16 \cdot 9)^{1/2} = 16^{1/2} \cdot 9^{1/2} = 4 \cdot 3 = 12$
4. $a^{-m} = \frac{1}{a^m}, a \neq 0$	$36^{-1/2} = \frac{1}{36^{1/2}} = \frac{1}{6}$
5. $\frac{a^m}{a^n} = a^{m-n}, a \neq 0$	$\frac{4^{5/2}}{4^{1/2}} = 4^{(5/2-1/2)} = 4^2 = 16$
6. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$	$\left(\frac{27}{64}\right)^{1/3} = \frac{27^{1/3}}{64^{1/3}} = \frac{3}{4}$

Example 1 Use the properties to simplify the expressions

a. $7^{1/4} \cdot 7^{1/2}$

$= 7^{1/4 + 1/2} = 7^{3/4}$

b. $\left(6^{1/2} \cdot 4^{1/3}\right)^2$

$= 6^{(1/2)2} \cdot 4^{(1/3)2}$

$= 6 \cdot 4^{2/3}$

c. $(4^5 \cdot 3^5)^{-1/5}$

$= \left[(4 \cdot 3)^5\right]^{-1/5}$

$= (12^5)^{-1/5} = 12^{-1} = \frac{1}{12}$

d. $\frac{5}{5^{1/3}} = 5^{1-1/3} = 5^{2/3}$

e. $\left(\frac{42^{1/3}}{6^{1/3}}\right)^2 = \left(\frac{42}{6}\right)^{1/3(2)} = 7^{2/3}$

Properties of Radicals

Product Property: $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

Quotient Property: $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

Example 3 Use the properties of radicals to simplify the expression

a. $\sqrt[3]{12} \cdot \sqrt[3]{18}$

$= \sqrt[3]{216}$

$= 6$

b. $\dfrac{\sqrt[4]{80}}{\sqrt[4]{5}} = \sqrt[4]{16} = 2$

Example 4 Write the expression in simplest form

the smallest 5th root is 32

a. $\sqrt[3]{135}$

$= \sqrt[3]{27} \cdot \sqrt[3]{5}$

$= 3\sqrt[3]{5}$

b. $\dfrac{\sqrt[5]{7}}{\sqrt[5]{8}} \left(\dfrac{\sqrt[5]{4}}{\sqrt[5]{4}}\right) = \dfrac{\sqrt[5]{28}}{\sqrt[5]{32}} = \dfrac{\sqrt[5]{28}}{2}$

Example 5 Add and subtract radicals and roots

a. $\sqrt[4]{10} + 7\sqrt[4]{10}$

$= (1+7)\sqrt[4]{10}$

$= 8\sqrt[4]{10}$

b. $2\left(8^{1/5}\right) + 10\left(8^{1/5}\right)$

$= 12\left(8^{1/5}\right)$

c. $\sqrt[3]{54} - \sqrt[3]{2}$

$= \sqrt[3]{27} \cdot \sqrt[3]{2} - \sqrt[3]{2}$

$= 3\sqrt[3]{2} - \sqrt[3]{2}$

$= (3-1)\sqrt[3]{2}$

$= 2\sqrt[3]{2}$

HW: (3-35 odd)

EXAMPLE 1
on p. 420
for Exs. 3–14

PROPERTIES OF RATIONAL EXPONENTS **Simplify the expression.**

3. $5^{3/2} \cdot 5^{1/2}$
4. $(6^{2/3})^{1/2}$
5. $3^{1/4} \cdot 27^{1/4}$
6. $\dfrac{9}{9^{-4/5}}$
7. $\dfrac{80^{1/4}}{5^{-1/4}}$
8. $\left(\dfrac{7^3}{4^3}\right)^{-1/3}$
9. $\dfrac{11^{2/5}}{11^{4/5}}$
10. $(12^{3/5} \cdot 8^{3/5})^5$
11. $\dfrac{120^{-2/5} \cdot 120^{2/5}}{7^{-3/4}}$
12. $\dfrac{64^{5/9} \cdot 64^{2/9}}{4^{3/4}}$
13. $(16^{5/9} \cdot 5^{7/9})^{-3}$
14. $\dfrac{13^{3/7}}{13^{5/7}}$

EXAMPLE 3
on p. 421
for Exs. 15–22

PROPERTIES OF RADICALS **Simplify the expression.**

15. $\sqrt{20} \cdot \sqrt{5}$
16. $\sqrt[3]{16} \cdot \sqrt[3]{4}$
17. $\sqrt[4]{8} \cdot \sqrt[4]{8}$
18. $(\sqrt[3]{3} \cdot \sqrt[4]{3})^{12}$
19. $\dfrac{\sqrt[5]{64}}{\sqrt[5]{2}}$
20. $\dfrac{\sqrt{3}}{\sqrt{75}}$
21. $\dfrac{\sqrt[4]{36} \cdot \sqrt[4]{9}}{\sqrt[4]{4}}$
22. $\dfrac{\sqrt[4]{8} \cdot \sqrt[4]{16}}{\sqrt[8]{2} \cdot \sqrt[8]{3}}$

EXAMPLE 4
on p. 422
for Exs. 23–31

23. ★ **MULTIPLE CHOICE** What is the simplest form of the expression $3\sqrt[4]{32} \cdot (-6\sqrt[4]{5})$?

Ⓐ $\sqrt[4]{10}$ Ⓑ $-18\sqrt[4]{10}$ Ⓒ $-36\sqrt[4]{10}$ Ⓓ $36\sqrt[8]{10}$

SIMPLEST FORM **Write the expression in simplest form.**

24. $\sqrt{72}$
25. $\sqrt[6]{256}$
26. $\sqrt[3]{108} \cdot \sqrt[3]{4}$
27. $5\sqrt[4]{64} \cdot 2\sqrt[4]{8}$
28. $\sqrt[3]{\dfrac{1}{6}}$
29. $\dfrac{3}{\sqrt[4]{144}}$
30. $\sqrt[6]{\dfrac{81}{4}}$
31. $\dfrac{\sqrt[3]{9}}{\sqrt[5]{27}}$

EXAMPLE 5
on p. 422
for Exs. 32–41

COMBINING RADICALS AND ROOTS **Simplify the expression.**

32. $2\sqrt[6]{3} + 7\sqrt[6]{3}$
33. $\frac{3}{5}\sqrt[3]{5} - \frac{1}{5}\sqrt[3]{5}$
34. $25\sqrt[5]{2} - 15\sqrt[5]{2}$
35. $\frac{1}{8}\sqrt[4]{7} + \frac{3}{8}\sqrt[4]{7}$
36. $6\sqrt[3]{5} + 4\sqrt[3]{625}$
37. $-6\sqrt[7]{2} + 2\sqrt[7]{256}$
38. $12\sqrt[4]{2} - 7\sqrt[4]{512}$
39. $2\sqrt[4]{1250} - 8\sqrt[4]{32}$
40. $5\sqrt[3]{48} - \sqrt[3]{750}$