6.3 – Perform Function Operations and Composition

KEY CONCEPT *For Your Notebook*

Operations on Functions

Let f and g be any two functions. A new function h can be defined by performing any of the four basic operations on f and g.

Operation	Definition	Example: $f(x) = 5x$, $g(x) = x + 2$
Addition	$h(x) = f(x) + g(x)$	$h(x) = 5x + (x + 2) = 6x + 2$
Subtraction	$h(x) = f(x) - g(x)$	$h(x) = 5x - (x + 2) = 4x - 2$
Multiplication	$h(x) = f(x) \cdot g(x)$	$h(x) = 5x(x + 2) = 5x^2 + 10x$
Division	$h(x) = \frac{f(x)}{g(x)}$	$h(x) = \frac{5x}{x + 2}$

The domain of h consists of the x-values that are in the domains of both f and g. Additionally, the domain of the quotient does not include x-values for which $g(x) = 0$.

Example 1 Add and Subtract Functions

Let $f(x) = 4x^{1/2}$ and $(x) = -9x^{1/2}$. Find the following:

a. $f(x) + g(x)$

$= 4x^{1/2} + (-9x^{1/2})$

$= (4 - 9)x^{1/2} = \boxed{-5x^{1/2}}$

b. $f(x) - g(x)$

$= 4x^{1/2} - (-9x^{1/2})$

$= (4 + 9)x^{1/2} = 13x^{1/2}$

c. the domains of $f + g$ and $f - g$

Non negative Real Numbers ($\mathbb{R}$)

Example 2 Multiply and divide functions

Let $f(x) = 6x$ and $(x) = x^{3/4}$. Find the following

a. $f(x)g(x)$

$= 6x(x^{3/4})$

$= 6x^{1+3/4} = 6x^{7/4}$

b. $\frac{f(x)}{g(x)}$

$= \frac{6x}{x^{3/4}} = 6x^{1-3/4} = 6x^{1/4}$

c. the domains of $f \cdot g$ and $\frac{f}{g}$

because $g(x) \neq 0$ for $\frac{f}{g}$ the domain is restricted to all positive $\mathbb{R}$

Another operation that can be performed with two functions is composition:

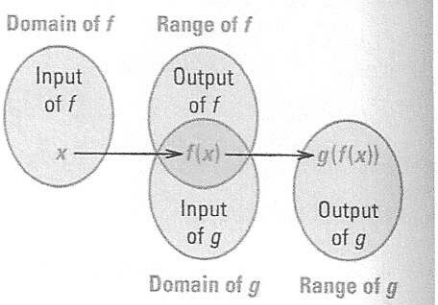
KEY CONCEPT *For Your Notebook*

Composition of Functions

The **composition** of a function g with a function f is:

$$h(x) = g(f(x))$$

The domain of h is the set of all x-values such that x is in the domain of f and $f(x)$ is in the domain of g.



Example 4 Standardized Test Practice

Let $f(x) = 2x - 7$ and $g(x) = x^2 + 4$. What is the value of $g(f(3))$?

Ⓐ −5 Ⓑ −3 Ⓒ 3 Ⓓ 5

$g(f(3)) = (2(3)-7)^2 + 4$

$= (-1)^2 + 4$

$= 5$ $\boxed{D}$

Example 5 Find compositions of functions

Let $f(x) = 4x^{-1}$ and $g(x)5x - 2$. Find the following:

a. $f(g(x))$

$= 4(5x-2)^{-1}$

$= \dfrac{4}{5x-2}$

b. $g(f(x))$

$= 5(4x^{-1}) - 2$

$= 20x^{-1} - 2$

$= \dfrac{20}{x} - 2$

c. $f(f(x))$

$= 4(4x^{-1})^{-1}$

$= 4(4^{-1}x)$

$= 4^{1-(-1)}x = 4^0 x = x$

~~d. the domain of each composition~~

HW: (3-37 odd)

EXAMPLE 1
on p. 428
for Exs. 3–11

ADD AND SUBTRACT FUNCTIONS **Let $f(x) = -3x^{1/3} + 4x^{1/2}$ and $g(x) = 5x^{1/3} + 4x^{1/2}$. Perform the indicated operation and state the domain.**

3. $f(x) + g(x)$
4. $g(x) + f(x)$
5. $f(x) + f(x)$
6. $g(x) + g(x)$
7. $f(x) - g(x)$
8. $g(x) - f(x)$
9. $f(x) - f(x)$
10. $g(x) - g(x)$

11. ★ **MULTIPLE CHOICE** What is $f(x) + g(x)$ if $f(x) = -7x^{2/3} - 1$ and $g(x) = 2x^{2/3} + 6$?

Ⓐ $5x^{2/3} - 5$ Ⓑ $-5x^{2/3} + 5$ Ⓒ $9x^{2/3} + 7$ Ⓓ $-9x^{2/3} - 7$

EXAMPLE 2
on p. 429
for Exs. 12–19

MULTIPLY AND DIVIDE FUNCTIONS **Let $f(x) = 4x^{2/3}$ and $g(x) = 5x^{1/2}$. Perform the indicated operation and state the domain.**

12. $f(x) \cdot g(x)$
13. $g(x) \cdot f(x)$
14. $f(x) \cdot f(x)$
15. $g(x) \cdot g(x)$
16. $\dfrac{f(x)}{g(x)}$
17. $\dfrac{g(x)}{f(x)}$
18. $\dfrac{f(x)}{f(x)}$
19. $\dfrac{g(x)}{g(x)}$

EXAMPLE 4
on p. 430
for Exs. 20–27

EVALUATE COMPOSITIONS OF FUNCTIONS **Let $f(x) = 3x + 2$, $g(x) = -x^2$, and $h(x) = \dfrac{x-2}{5}$. Find the indicated value.**

20. $f(g(-3))$
21. $g(f(2))$
22. $h(f(-9))$
23. $g(h(8))$
24. $h(g(5))$
25. $f(f(7))$
26. $h(h(-4))$
27. $g(g(-5))$

EXAMPLE 5
on p. 430
for Exs. 28–38

FIND COMPOSITIONS OF FUNCTIONS **Let $f(x) = 3x^{-1}$, $g(x) = 2x - 7$, and $h(x) = \dfrac{x+4}{3}$. Perform the indicated operation and state the domain.**

28. $f(g(x))$
29. $g(f(x))$
30. $h(f(x))$
31. $g(h(x))$
32. $h(g(x))$
33. $f(f(x))$
34. $h(h(x))$
35. $g(g(x))$

ERROR ANALYSIS **Let $f(x) = x^2 - 3$ and $g(x) = 4x$. *Describe* and correct the error in the composition.**

36.

$$f(g(x)) = f(4x)$$
$$= (x^2 - 3)(4x)$$
$$= 4x^3 - 12x$$

37.

$$g(f(x)) = g(x^2 - 3)$$
$$= 4x^2 - 3$$