6.4 – Use Inverse Functions

An inverse relation interchanges the input one output values of the original relation. This means the domain and range are also interchanged. The graph of an inverse relation is a reflection of the graph of the original relation.

To find the inverse of a relation given by an equation in x and y, switch the roles of x and y and solve for y.

Example 1 Find an equation for the inverse of the relation $y = 3x - 5$

$y = 3x - 5$

$x = 3y - 5$

$+5 \quad +5$

$\frac{x+5}{3} = \frac{3y}{3}$

$y = \frac{1}{3}x + \frac{5}{3}$

Inverse Functions – Functions f and g are inverses of each other provided:

$f(g(x)) = x$ and $g(f(x)) = x$

The inverse of function f is denoted as f^{-1} and is pronounced "f inverse"

Example 2 Verify that $f(x) = 3x - 5$ and $f^{-1}(x) = \frac{1}{3}x + \frac{5}{3}$

$f(f^{-1}(x)) = 3\left(\frac{1}{3}x + \frac{5}{3}\right) - 5$

$= x + 5 - 5$

$= x$ ✓

$f^{-1}(f(x)) = \frac{1}{3}(3x - 5) + \frac{5}{3}$

$= x - \frac{5}{3} + \frac{5}{3}$

$= x$ ✓

Example 4 Find the inverse of a power function

Find the inverse of $f(x) = x^2$

$f(x) = x^2$

$y = x^2$

$\sqrt{x} = \sqrt{y^2}$

$y = \pm\sqrt{x} \rightarrow f^{-1}(x) = \pm\sqrt{x}$

HW: (3-27 odd)

EXAMPLE 1
on p. 438
for Exs. 3–13

INVERSE RELATIONS **Find an equation for the inverse relation.**

3. $y = 4x - 1$
4. $y = -2x + 5$
5. $y = 7x - 6$
6. $y = 10x - 28$
7. $y = 12x + 7$
8. $y = -18x - 5$
9. $y = 5x + \frac{1}{3}$
10. $y = -\frac{2}{3}x + 2$
11. $y = -\frac{3}{5}x + \frac{7}{5}$

ERROR ANALYSIS ***Describe*** **and correct the error in finding the inverse of the relation.**

12.

$$y = 6x - 11$$
$$x = 6y - 11$$
$$x + 11 = 6y$$
$$\frac{x}{6} + 11 = y$$

13.

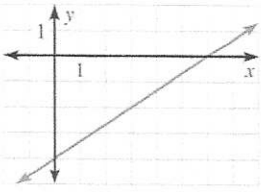
$$y = -x + 3$$
$$-x = y + 3$$
$$-x - 3 = y$$

EXAMPLE 2
on p. 439
for Exs. 15–21

VERIFYING INVERSE FUNCTIONS **Verify that f and g are inverse functions.**

15. $f(x) = x + 4, g(x) = x - 4$
16. $f(x) = 2x + 3, g(x) = \frac{1}{2}x - \frac{3}{2}$
17. $f(x) = \frac{1}{4}x^3, g(x) = (4x)^{1/3}$
18. $f(x) = \frac{1}{5}x - 1, g(x) = 5x + 5$
19. $f(x) = 4x + 9, g(x) = \frac{1}{4}x - \frac{9}{4}$
20. $f(x) = 5x^2 - 2, x \geq 0; g(x) = \left(\frac{x+2}{5}\right)^{1/2}$

21. ★ **MULTIPLE CHOICE** What is the inverse of the function whose graph is shown?



Ⓐ $g(x) = \frac{3}{2}x - 6$ Ⓑ $g(x) = \frac{3}{2}x + 6$

Ⓒ $g(x) = \frac{2}{3}x - 6$ Ⓓ $g(x) = \frac{3}{2}x + 12$

EXAMPLE 4
on p. 440
for Exs. 22–28

INVERSES OF POWER FUNCTIONS **Find the inverse of the power function.**

22. $f(x) = x^7$
23. $f(x) = 4x^4, x \geq 0$
24. $f(x) = -10x^6, x \leq 0$
25. $f(x) = 32x^5$
26. $f(x) = -\frac{2}{5}x^3$
27. $f(x) = \frac{16}{25}x^2, x \leq 0$

28. ★ **MULTIPLE CHOICE** What is the inverse of $f(x) = -\frac{1}{64}x^3$?

Ⓐ $g(x) = -4x^3$ Ⓑ $g(x) = 4\sqrt[3]{x}$ Ⓒ $g(x) = -4\sqrt[3]{x}$ Ⓓ $g(x) = \sqrt[3]{-4x}$