6.6 – Solving Radical Equations

Step 1	Isolate the radical on one side of the equation, if necessary
Step 2	Raise each side of the equation to the same power to eliminate the radical and obtain a linear, quadratic, or other polynomial equation
Step 3	Solve the polynomial equation using techniques you learned in previous chapters. Check your solution

Example 1 Solve $\sqrt[3]{2x+7} = 3$

$\left(\sqrt[3]{2x+7}\right)^3 = 3^3$

$2x+7 = 27$

$-7 \quad -7$

$\frac{2x}{2} = \frac{20}{2}$

$x = 10$

$\sqrt[3]{2(10)+7} = 3$

$\sqrt[3]{27} = 3$

$3 = 3$ ✓

Example 2 In a hurricane, the mean sustained wind velocity v (in meters per second) is given by: $v(p) = 6.3\sqrt{1013 - p}$ where p is the air pressure (in millibars) at the center of the hurricane. Estimate the air pressure at the center of a hurricane when the mean sustained wind velocity is 54.5 meters per second.

$\frac{54.5}{6.3} = \frac{6.3\sqrt{1013-p}}{6.3}$

$8.65^2 = \left(\sqrt{1013-p}\right)^2$

$74.84 = 1013 - p$

$-1013 \quad -1013$

$p = 938.16$

Example 3 Standardized Test Prep

What is the solution of the equation $4x^{3/2} = 108$?

Ⓐ 3 Ⓑ 6 Ⓒ 9 Ⓓ 27

$\frac{4x^{3/2}}{4} = \frac{108}{4}$

$(x^{3/2})^{2/3} = 27^{2/3}$

$x = \sqrt[3]{27^2}$

$x = 9$ C

Example 4 Solve $(x+2)^{3/4} - 1 = 7$

$+1 \quad +1$

$\left[(x+2)^{3/4}\right]^{4/3} = 8^{4/3}$

$x+2 = (\sqrt[3]{8})^4$

$x+2 = 2^4$

$x+2 = 16 \rightarrow x = 14$

$-2 \quad -2$

Example 5 Solve an extraneous solution = a solution that doesn't check

Solve $x + 1 = \sqrt{7x+15}$

$(x+1)^2 = (\sqrt{7x+15})^2$

$x^2 + 2x + 1 = 7x + 15$

$-7x-15 \quad -7x-15$

$x^2 - 5x - 14 = 0$

$(x-7)(x+2) = 0$

$x = 7 \quad x = -2$

$x = 7$

$7+1 = \sqrt{7(7)+15}$

$8 = \sqrt{64}$

$8 = 8$ ✓

$x = -2$

$-2+1 = \sqrt{7(-2)+15}$

$-1 = \sqrt{1}$

$-1 \neq 1$

extraneous

$x = -2$

Example 6 Solve with TWO radicals

Solve $\sqrt{x+2} + 1 = \sqrt{3-x}$

$(\sqrt{x+2}+1)^2 = (\sqrt{3-x})^2$

FOIL

$(x+2) + 2\sqrt{x+2} + 1 = 3 - x$

$x + 3 + 2\sqrt{x+2} = 3 - x$

$-x \quad -3 \qquad\qquad -3 - x$

$\frac{2\sqrt{x+2}}{2} = \frac{-2x}{2}$

$\sqrt{x+2} = -x$

$(\sqrt{x+2})^2 = (-x)^2$

$x + 2 = x^2$

$-x \quad -2 \qquad -x - 2$

$0 = x^2 - x - 2$

$0 = (x-2)(x+1)$

$x = 2 \quad x = -1$

extraneous