

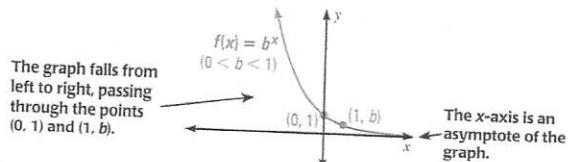
7.2 – Graph Exponential Decay Functions

KEY CONCEPT

For Your Notebook

Parent Function for Exponential Decay Functions

The function $f(x) = b^x$, where $0 < b < 1$, is the parent function for the family of exponential decay functions with base b . The general shape of the graph of $f(x) = b^x$ is shown below.

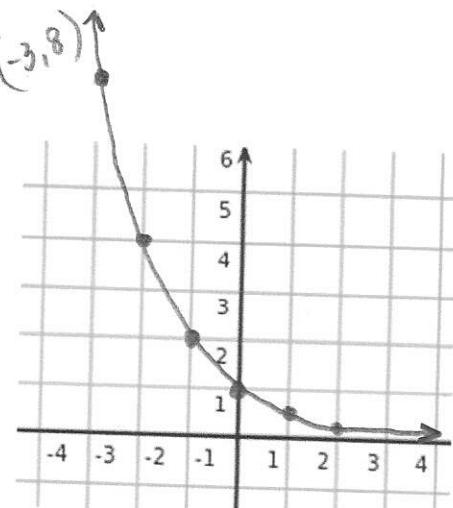


The domain of $f(x) = b^x$ is all real numbers. The range is $y > 0$.

Example 1 Graph $y = b^x$ for $0 < b < 1$

$$\text{Graph } y = \left(\frac{1}{2}\right)^x$$

x	-3	-2	-1	0	1	2
y	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$



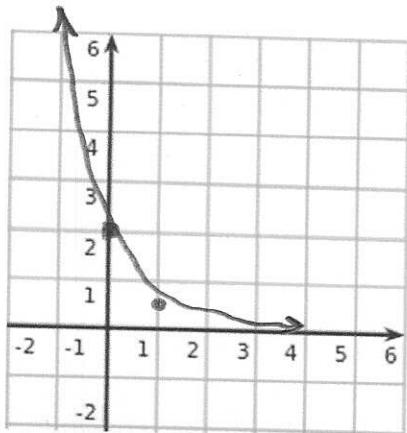
$$\begin{aligned} x = -3 & \quad y = \left(\frac{1}{2}\right)^{-3} = 2^3 \\ x = -2 & \quad y = \left(\frac{1}{2}\right)^{-2} = 2^2 \\ x = -1 & \quad y = \left(\frac{1}{2}\right)^{-1} = 2^1 \\ x = 0 & \quad y = \left(\frac{1}{2}\right)^0 = 1 \\ x = 1 & \quad y = \left(\frac{1}{2}\right)^1 \\ x = 2 & \quad y = \left(\frac{1}{2}\right)^2 = \frac{1}{2^2} \end{aligned}$$

Example 2 Graph $y = ab^x$ for $0 < b < 1$

Graph the function.

a. $y = 2\left(\frac{1}{4}\right)^x$

plot $x=0$ & $x=1$
then sketch
the curve

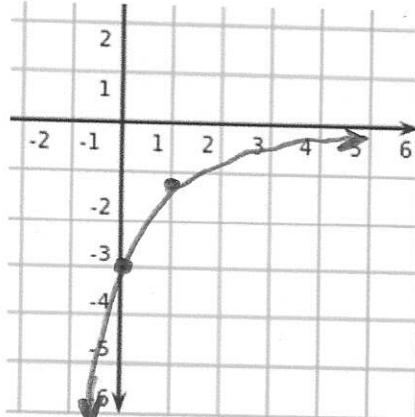


$$\begin{aligned} x = 0 & \quad y = 2\left(\frac{1}{4}\right)^0 = 2(1) \\ y &= 2 \end{aligned}$$

x	0	1
y	2	$\frac{1}{2}$

$$\begin{aligned} x = 1 & \quad y = 2\left(\frac{1}{4}\right)^1 = 2\left(\frac{1}{4}\right) = \frac{2}{4} \end{aligned}$$

b. $y = -3\left(\frac{2}{5}\right)^x$



$$\begin{aligned} x = 0 & \quad y = -3\left(\frac{2}{5}\right)^0 = -3(1) \\ y &= -3 \end{aligned}$$

x	0	1
y	-3	$-\frac{6}{5}$

$$\begin{aligned} x = 1 & \quad y = -3\left(\frac{2}{5}\right)^1 = -3\left(\frac{2}{5}\right) \\ y &= -\frac{6}{5} \end{aligned}$$

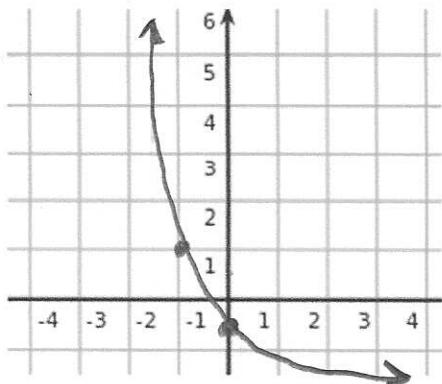
Example 3 Graph $y = ab^{x-h} + k$ for $0 < b < 1$

$$\text{Graph } y = 3\left(\frac{1}{2}\right)^{x+1} - 2$$

- ① find $x=0$ & $x=1$ for $y = 3\left(\frac{1}{2}\right)^x$
- ② translate those pts. for (h, k)

$$\begin{aligned} \textcircled{1} \quad x=0 & \quad y = 3\left(\frac{1}{2}\right)^0 = 3 \\ x=1 & \quad y = 3\left(\frac{1}{2}\right)^1 = \frac{3}{2} \end{aligned}$$

$$\textcircled{2} \quad (h, k) = (-1, -2) \quad (0, 3) \Rightarrow \boxed{(-1, 1)} \neq (1, \frac{3}{2}) \Rightarrow \boxed{(0, -\frac{1}{2})}$$



Example 4 Modeling

SNOWMOBILES A new snowmobile costs \$4200. The value of the snowmobile decreases by 10% each year.

- Write an exponential decay model giving the snowmobile's value y (in dollars) after t years. Estimate the value after 3 years.



$$y = a(1-r)^t$$

$$a = 4200$$

$$r = 10\% \text{ or } .1$$

$$y = 4200(1 - .1)^t$$

$$= 4200(0.9)^t$$

after 3 yrs

$$y = 4200(0.9)^3$$

$$y = \$3061.80$$

HW: (3-10), (16-18), (30)

CLASSIFYING FUNCTIONS Tell whether the function represents *exponential growth* or *exponential decay*.

3. $f(x) = 3\left(\frac{3}{4}\right)^x$

4. $f(x) = 4\left(\frac{5}{2}\right)^x$

5. $f(x) = \frac{2}{7} \cdot 4^x$

6. $f(x) = 25(0.25)^x$

EXAMPLES

and 2

pp. 486-487
for Exs. 7-15

GRAPHING FUNCTIONS Graph the function.

7. $y = \left(\frac{1}{4}\right)^x$

8. $y = \left(\frac{1}{3}\right)^x$

9. $f(x) = 2\left(\frac{1}{5}\right)^x$

10. $y = -(0.2)^x$

11. $y = -4\left(\frac{1}{3}\right)^x$

12. $g(x) = 2(0.75)^x$

13. $y = \left(\frac{3}{5}\right)^x$

14. $h(x) = -3\left(\frac{3}{8}\right)^x$

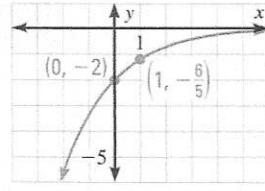
15. ★ **MULTIPLE CHOICE** The graph of which function is shown?

(A) $y = 2\left(-\frac{3}{5}\right)^x$

(B) $y = -2\left(\frac{3}{5}\right)^x$

(C) $y = -2\left(\frac{2}{5}\right)^x$

(D) $y = 2\left(-\frac{2}{5}\right)^x$



EXAMPLE 3

pp. 487

Exs. 16-25

TRANSLATING GRAPHS Graph the function. State the domain and range.

16. $y = \left(\frac{1}{3}\right)^x + 1$

17. $y = -\left(\frac{1}{2}\right)^{x-1}$

18. $y = 2\left(\frac{1}{3}\right)^{x+1} - 3$

19. $y = \left(\frac{2}{3}\right)^{x-4} - 1$

20. $y = 3(0.25)^x + 3$

21. $y = \left(\frac{1}{3}\right)^{x-2} + 2$

22. $f(x) = -3\left(\frac{1}{4}\right)^{x-1}$

23. $g(x) = 6\left(\frac{1}{2}\right)^{x+5} - 2$

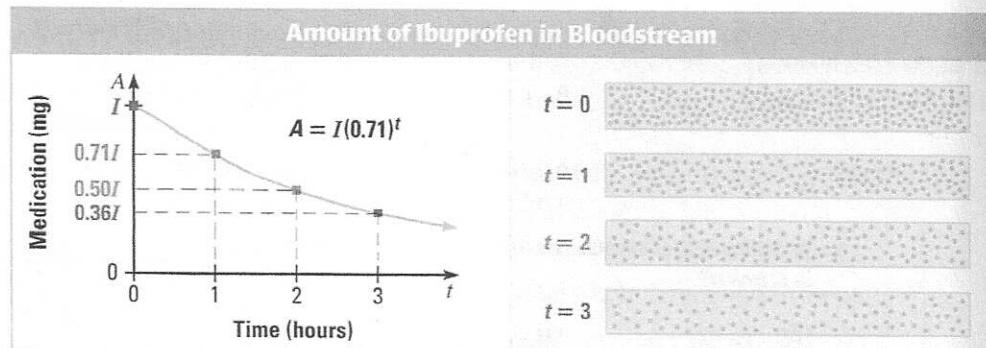
24. $h(x) = 4\left(\frac{1}{2}\right)^{x+1}$

EXAMPLE 4

on p. 488

for Exs. 30-31

30. **MEDICINE** When a person takes a dosage of I milligrams of ibuprofen, the amount A (in milligrams) of medication remaining in the person's bloodstream after t hours can be modeled by the equation $A = I(0.71)^t$.



Find the amount of ibuprofen remaining in a person's bloodstream for the given dosage and elapsed time since the medication was taken.

- a. Dosage: 200 mg
Time: 1.5 hours
- b. Dosage: 325 mg
Time: 3.5 hours
- c. Dosage: 400 mg
Time: 5 hours