7.3 – Use Functions involving e

The Euler Number (e) – or the natural base e represents the number ≈ 2.718281828

The expression $\left(1+\frac{1}{n}\right)^n$ approaches e as the value of n increases. For example:

n	10	100	1,000	10,000	100,000	1,000,000
$\left(1+\frac{1}{n}\right)^n$	2.59374	2.70481	2.71692	2.71815	2.71827	2.71828

Example 1 Simplify natural base expressions

a. $e^2 \cdot e^5$

$= e^{2+5}$

$= e^7$

b. $\frac{12e^4}{3e^3}$

$= \left(\frac{12}{3}\right)e^{4-3}$

$= 4e^1$

c. $(5e^{-3x})^2$

$= 5^2 e^{-3x(2)}$

$= 25e^{-6x}$

Example 2 Evaluate natural base expressions

a. $e^4 \approx 54.60$

b. $e^{-0.09} \approx 0.91$

Natural Base Exponential Function

$$y = ae^{rx}$$

Growth: $a > 0$ *and* $r > 0$

Decay: $a > 0$ *and* $r < 0$

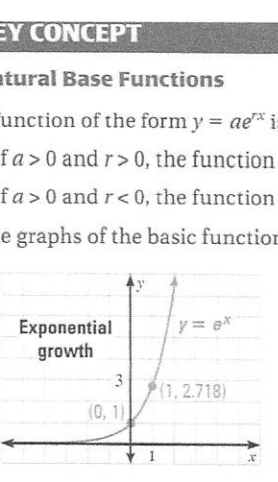
KEY CONCEPT *For Your Notebook*

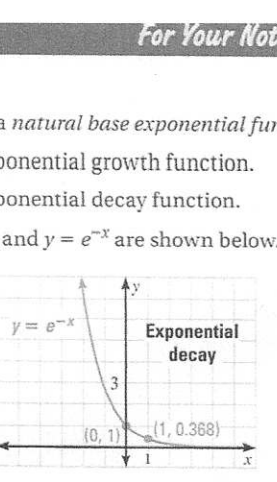
Natural Base Functions

A function of the form $y = ae^{rx}$ is called a *natural base exponential function.*

- If $a > 0$ and $r > 0$, the function is an exponential growth function.
- If $a > 0$ and $r < 0$, the function is an exponential decay function.

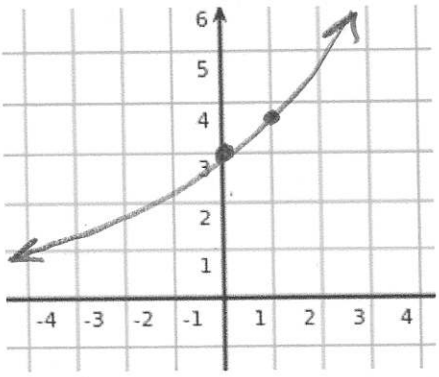
The graphs of the basic functions $y = e^x$ and $y = e^{-x}$ are shown below.





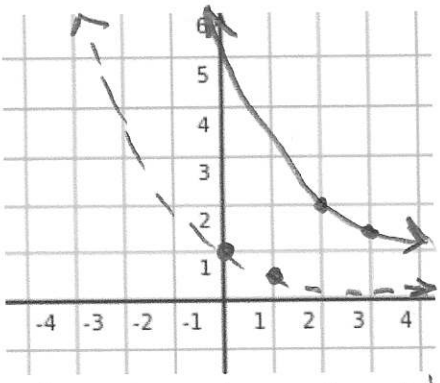
Example 3 Graph natural base functions

a. $y = 3e^{0.25x}$ $a > 0$ $r > 0$ $\Rightarrow$ growth



when $x = 0$ $y = 3$ $(0, 3)$, when $x = 1$ $y \approx 3.85$

b. $y = e^{-0.75(x-2)} + 1$ $a > 0$ $r < 0$ $\Rightarrow$ decay



① use $y = e^{-.75x}$

② when $x = 0$ $y = 1$ $(0, 1)$

③ when $x = 1$ $y \approx 0.47$ $(1, .47)$

⑤ $(0, 1) = (2, 2)$ $(1, .47) = (3, 1.47)$
$+2$ $+1$ $+2$ $+1$

④ $(h, k) = (2, 1)$

⑤ translate the 2 pts. from steps 2 & 3 by (h, k) in step 4

Continuously Compounded Interest

7.1 Formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$, so as the number of times interest is compounded (n) increases infinitely, we are left with the expression for e from the beginning of the lesson. And the new Formula for Continuously Compounded Interest is:

$$A = Pe^{rt}$$

Example 5 Model continuously compounded interest

You deposit \$4000 in an account the pays 6% annual interest compounded continuously. What is the balance after 1 year?

$A = Pe^{rt}$ $P = 4000$

$r = 6\% = .06$

$t = 1$

$A = 4000e^{.06(1)}$

$A = \$4247.35$

HW: (3-14)(27-44)(57-58)

EXAMPLE 1 on p. 492 for Exs. 3–18

SIMPLIFYING EXPRESSIONS **Simplify the expression.**

3. $e^3 \cdot e^4$
4. $e^{-2} \cdot e^6$
5. $(2e^{3x})^3$
6. $(2e^{-2})^{-4}$
7. $(3e^{5x})^{-1}$
8. $e^x \cdot e^{-3x} \cdot e^4$
9. $\sqrt{9e^6}$
10. $e^x \cdot 5e^{x+3}$
11. $\dfrac{3e}{e^x}$
12. $\dfrac{4e^x}{e^{4x}}$
13. $\sqrt[3]{8e^{9x}}$
14. $\dfrac{6e^{4x}}{8e}$

EXAMPLE 2 on p. 492 for Exs. 19–30

EVALUATING EXPRESSIONS **Use a calculator to evaluate the expression.**

19. e^3
20. $e^{-3/4}$
21. $e^{2.2}$
22. $e^{1/2}$
23. $e^{-2/5}$
24. $e^{4.3}$
25. e^7
26. e^{-4}
27. $2e^{-0.3}$
28. $5e^{2/3}$
29. $-6e^{2.4}$
30. $0.4e^{4.1}$

GROWTH OR DECAY **Tell whether the function is an example of *exponential growth* or *exponential decay*.**

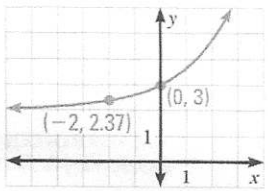
31. $f(x) = 3e^{-x}$
32. $f(x) = \frac{1}{3}e^{4x}$
33. $f(x) = e^{-4x}$
34. $f(x) = \frac{3}{5}e^x$
35. $f(x) = \frac{1}{4}e^{-5x}$
36. $f(x) = e^{3x}$
37. $f(x) = 2e^{4x}$
38. $f(x) = 4e^{-2x}$

EXAMPLE 3 on p. 493 for Exs. 39–50

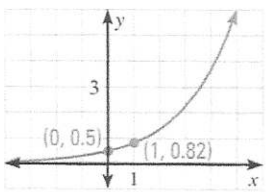
MATCHING GRAPHS **Match the function with its graph.**

39. $y = 0.5e^{0.5x}$
40. $y = 2e^{0.5x}$
41. $y = e^{0.5x} + 2$

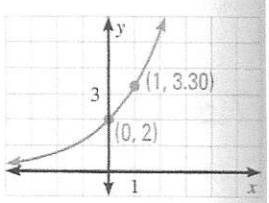
A.



B.



C.



GRAPHING FUNCTIONS **Graph the function. State the domain and range.**

42. $y = e^{-2x}$
43. $y = 3e^x$
44. $y = 0.5e^x$
45. $y = 2e^{-3x} - 1$
46. $y = 2.5e^{-0.5x} + 2$
47. $y = 0.6e^{x-2}$
48. $f(x) = \frac{1}{2}e^{x+3} - 2$
49. $g(x) = \frac{4}{3}e^{x-1} + 1$
50. $h(x) = e^{-2(x+1)} - 3$

EXAMPLE 5 on p. 495 for Exs. 57–58

57. **FINANCE** You deposit \$2000 in an account that pays 4% annual interest compounded continuously. What is the balance after 5 years?

58. **FINANCE** You deposit \$800 in an account that pays 2.65% annual interest compounded continuously. What is the balance after 12.5 years?