Factoring Quadratic Trinomials

Standard Form of a Quadratic Trinomial

$$y = ax^2 + bx + c$$

a = coefficient of the x^2 term

b = coefficient of the x term

c = the constant (no variable present)

When factoring $ax^2 + bx + c$, find two factors of c whose sum is equal to b

Example 1 Factor $x^2 + 9x + 18$

Factors of 18	Sum of 9
18 · 1	19
9 · 2	11
6 · 3	9

$\boxed{(x+6)(x+3)}$

$x^2 + 6x + 8$

Factors of 8	Sum of 6
8 · 1	9
2 · 4	6

$\boxed{(x+2)(x+4)}$

When c is positive, the factors have the same sign

If b is positive, the signs of the factors are positive

If b is negative, the signs of the factors are negative

$x^2 + bx + c$

$(x + m)(x + n)$

$x^2 - bx + c$

$(x - m)(x - n)$

Example 2 Factor

$x^2 + 5x + 6$ $(x+\ )(x+\ )$

$m \cdot n = 6$

$m + n = 5$

$m = 2 \quad n = 3$

$\boxed{(x+2)(x+3)}$

$x^2 - 10x + 16$ $(x-2)(x-8)$

$m \cdot n = 16$

$m + n = -10$

$m = -2 \quad n = -8$

$(x-2)(x-8)$

When c is negative, the factors will have opposite signs

If b is positive, the larger factor will also be positive

If b is negative, the larger factor will be negative

$x^2 + bx - c$ $\qquad\qquad$ $x^2 - bx - c$

$(x+m)(x-n)$ $\qquad$ $\leftarrow m > n \rightarrow$ $\qquad$ $(x-m)(x+n)$

Example 3 Factor

$x^2 + 7x - 18$ $\quad (x+9)(x-2)$

$m \cdot n = -18$

$m + n = 7$

$m = 9 \quad n = -2$

$\boxed{(x+9)(x-2)}$

$x^2 - 5x - 24$

$m \cdot n = -24$

$m + n = -5$

$m = -8 \quad n = 3$

$\boxed{(x-8)(x+3)}$

Factoring when $a \neq 1$

Step 1	Multiply a and c
Step 2	Find factors of ac with a sum of b
Step 3	Rewrite as a 4-term polynomial using factors from step 2
Step 4	Factor by grouping or boxing

**note: the same rules with signs will also apply*

Example 4 Factor

a. $3x^2 + 16x + 5$

$\boxed{S1}$ $3 \cdot 5 = 15$

$\boxed{S2}$ $15 \cdot 1 = 15$

$15 + 1 = 16$

$\boxed{S3}$ $3x^2 + 15x + 1x + 5$

$\boxed{S4}$ Grouping

$(3x^2 + 15x) + (1x + 5)$

$3x(x+5) + 1(x+5)$

$\boxed{(3x+1)(x+5)}$

b. $5x^2 - 9x - 2$

$\boxed{S1}$ $5 \cdot -2 = -10$

$\boxed{S2}$ $-10 \cdot 1 = -10$

$-10 + 1 = -9$

$\boxed{S3}$ $5x^2 - 10x + 1x - 2$

$\boxed{S4}$ Boxing

$5x^2$	$-10x$	$5x$
$1x$	-2	$+1$
x	-2	

$\boxed{(5x+1)(x-2)}$

Factoring Trinomials When $a > 1$

Factor by Grouping. Use a separate sheet of paper.

1. $3x^2 + 16x + 5$
2. $6x^2 - 11x - 2$
3. $5x^2 - 9x - 2$
4. $6x^2 - 11x - 10$

Factor by Boxing.

5. $10x^2 + 17x + 6$
6. $14x^2 - 15x + 4$
7. $8x^2 + 2x - 3$
8. $2x^2 + 19x - 10$

FAST METHOD

$3x^2 + 16x + 5$

① Mult. ac $3(5) = 15$

② Bring down $x^2 + 16x$ and add ac

$x^2 + 16x + 15$

③ factor

$(x+15)(x+1)$

④ divide a into factors

$(x + \frac{15}{3})(x + \frac{1}{3})$

⑤ Simplify

$(x+5)(x + \frac{1}{3})$

⑥ if you're left with a fraction, the denominator becomes the coefficient of x in that factor.

$(x+5)(3x+1)$