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P 10

~~29~~ - Logarithmic Functions

A Logarithmic Function is an inverse function to an Exponential Function, therefore logarithms are exponents

Relating Logarithms to exponents

$$\log_b x = y \quad \text{if and only if} \quad b^y = x$$

Example 1 Evaluating Logarithms

a. $\log_3 81 = x$

$$3^x = 81 \quad x = 4$$

b. $\log_5 \sqrt{5}$

$$5^x = \sqrt{5} \quad x = \frac{1}{2}$$

c. $\log_7 \frac{1}{49}$

$$7^x = \frac{1}{49} \quad x = -2$$

d. $\log_2 2$

$$2^x = 1 \quad x = 0$$

Basic Logarithmic Properties $b > 0, b \neq 1$

$\log_b 1 = 0$

$\log_b b = 1$

$\log_b b^x = x$

$b^{\log_b x} = x, x > 0$

Example 2 Applying Logarithmic Properties

a. $\log_5 125$

$$\log_5 5^3 = 3$$

b. $12^{\log_{12} 4.7} = 4.7$

c. $\log_9 81$

$$\log_9 9^2 = 2$$

d. $3^{\log_3 1} = 1$

Common Logarithms

logarithms with a base of 10

$\log 1 = 0$

$\log 10 = 1$

$\log 10^x = x$

$10^{\log x} = x, x > 0$

Example 3 Evaluating Common Logarithms

a. $\log 0.001$

$$-3$$

b. $10^{\log 5}$

$$5$$

$$c. \log 26 = 1.415$$

$$d. \log(-5) \text{ not real}$$

Natural Logarithms logarithms with a base of e
(natural logarithms are written as \ln)

$$e = 2.71828\dots$$

Basic Properties of natural logarithms

$$\ln 1 = 0$$

$$\ln e = 1$$

$$\ln e^x = x$$

$$e^{\ln x} = x, x > 0$$

Example 4 Evaluating Natural Logarithms

$$a. \ln e^{0.73} = 0.73$$

$$b. \ln(-5) \text{ not real}$$

$$e^x = -5$$

$$c. e^{\ln 6} = 6$$

$$d. \ln 4 = 1.386$$

Evaluate each expression. (Examples 1-4)

1. $\log_2 8$

2. $\log_{10} 10$

3. $\log_6 \frac{1}{36}$

4. $4^{\log_4 1}$

5. $\log_{11} 121$

6. $\log_2 2^3$

7. $\log_{\sqrt{9}} 81$

8. $\log 0.01$

9. $\log 42$

10. $\log_x x^2$

11. $\log 5275$

12. $\ln e^{-14}$

13. $3 \ln e^4$

14. $\ln(5 - \sqrt{6})$

15. $\log_{36} \sqrt[5]{6}$

16. $4 \ln(7 - \sqrt{2})$

17. $\log 635$

18. $\frac{\ln 2}{\ln 7}$

19. $\ln(-6)$

20. $\ln\left(\frac{1}{e^{12}}\right)$

21. $\ln 8$

22. $\log_{\sqrt{4}} 64$

23. $\frac{7}{\ln e}$

24. $\log 1000$