

P 11  
~~P 10~~ **Properties of Logarithms**

Do you remember exponential properties

$$b^x \cdot b^y = b^{x+y}$$

$$\frac{b^x}{b^y} = b^{x-y}$$

$$(b^x)^y = b^{xy}$$

Logarithmic Properties

Product Property  $\log_b xy = \log_b x + \log_b y$

Quotient Property  $\log_b \frac{x}{y} = \log_b x - \log_b y$

Power Property  $\log_b x^p = p \log_b x$

Example 1  $\log_4 15 = \log_4 5 + \log_4 3$

Example 2  $\log_4 \left(\frac{5}{3}\right) = \log_4 5 - \log_4 3$

Example 3  $\log_4 (t^2) = 2 \log_4 t$

Example 4  $\log_4 \left(\frac{xy}{z^3}\right) = \log_4 x + \log_4 y - 3 \log_4 z$

Example 5  $\log_4 3 + 2 \log_4 x - \log_4 y = \log_4 \left(\frac{3x^2}{y}\right)$

In Exercises 1–12, express each as a sum, difference, or multiple of logarithms. See Example 2.

1.  $\log_5 33$

2.  $\log_3 14$

3.  $\log_7 \left(\frac{2}{3}\right)$

4.  $\log_3 \left(\frac{2}{11}\right)$

5.  $\log_2 (a^3)$

6.  $\log_8 (n^5)$

7.  $\log_6 abc$

8.  $\log_2 \left(\frac{xy}{z^2}\right)$

9.  $\log_5 \sqrt[4]{y}$

10.  $\log_4 \sqrt[3]{x}$

11.  $\log_2 \left(\frac{\sqrt{x}}{a^2}\right)$

12.  $\log_3 \left(\frac{\sqrt[3]{y}}{7}\right)$

In Exercises 13–20, express each as the logarithm of a single quantity. See Example 3.

13.  $\log_b a + \log_b c$

14.  $\log_2 3 + \log_2 x$

15.  $\log_5 9 - \log_5 3$

16.  $\log_8 V - \log_8 R$

17.  $\log_b x^2 - \log_b \sqrt{x}$

18.  $\log_4 3^3 + \log_4 9$

19.  $2 \log_e 2 + 3 \log_e n$

20.  $\frac{1}{2} \log_b a - 2 \log_b 5$

In Exercises 21–28, determine the exact value of each of the given logarithms.

21.  $\log_2 \left(\frac{1}{32}\right)$

22.  $\log_3 \left(\frac{1}{81}\right)$

23.  $\log_2 (2^{2.5})$

24.  $\log_5 (5^{0.1})$

25.  $\log_7 \sqrt{7}$

26.  $\log_6 \sqrt[3]{6}$

27.  $\log_3 \sqrt[3]{27}$

28.  $\log_5 \sqrt[3]{25}$

