

P12 - Inverse Functions

If a function has an inverse that is also a function, it is called a *one to one* function

All *one to one* functions pass both the vertical and horizontal line tests for the graph of the function f

The inverse of a *one to one* function is denoted as: $f^{-1}(x)$

To find the inverse of a function:

Step 1	Replace $f(x)$ with y
Step 2	Interchange x and y
Step 3	Solve for y
Step 4	Replace y with $f^{-1}(x)$

Example 1 Find the inverse function

a. $f(x) = \frac{x-1}{x+2}$

$$y = \frac{x-1}{x+2}$$

$$x = \frac{y-1}{y+2}$$

$$x(y+2) = y-1$$

$$xy + 2x = y-1$$

$$xy - y = -2x - 1$$

$$y(x-1) = -2x - 1$$

$$y = \frac{-2x-1}{x-1}$$

$$f^{-1}(x) = \frac{-2x-1}{x-1}$$

b. $f(x) = \sqrt{x-4}$

$$y = \sqrt{x-4}$$

$$x = \sqrt{y-4}$$

$$x^2 = y-4$$

$$y = x^2 + 4$$

$$f^{-1}(x) = x^2 + 4$$

c. $f(x) = \frac{x+7}{x}$

$$y = \frac{x+7}{x}$$

$$x = \frac{y+7}{y}$$

$$xy = y+7$$

$$xy - 7 =$$

$$xy - y = 7$$

$$y(x-1) = 7$$

$$y = \frac{7}{x-1}$$

$$f^{-1}(x) = \frac{7}{x-1}$$

Find $f^{-1}(x)$

13. $g(x) = -3x^4 + 6x^2 - x$

14. $f(x) = 4x^5 - 8x^4$

15. $h(x) = x^7 + 2x^3 - 10x^2$

16. $f(x) = \sqrt{x+8}$

17. $f(x) = \sqrt{6-x^2}$

18. $f(x) = |x-6|$

19. $f(x) = \frac{4-x}{x}$

20. $g(x) = \frac{x-6}{x}$

21. $f(x) = \frac{6}{\sqrt{8-x}}$

22. $g(x) = \frac{7}{\sqrt{x+3}}$

23. $f(x) = \frac{6x+3}{x-8}$

24. $h(x) = \frac{x+4}{3x-5}$

25. $g(x) = |x+1| + |x-4|$

$$\textcircled{24} \quad h(x) = \frac{x+4}{3x-5} \quad x = \frac{y+4}{3y-5}$$

$$x(3y-5) = y+4$$

$$3xy - 5x = y+4$$

$$3xy - y = 5x+4$$

$$y(3x-1) = 5x+4$$

$$y = \frac{5x+4}{3x-1}$$

$$f^{-1}(x) = \frac{5x+4}{3x-1}$$