P.2 – INTERCEPTS AND SYMMETRY

Solution points that are especially useful in graphing are called intercepts. The point $(a, 0)$ is referred to as the x-intercept, or the point where the graph crosses the x-axis. The point $(0, b)$ is referred to as the y-intercept, or the point where the graph crosses the y-axis.

To solve the x-intercept(s), let y be zero and solve the equation for x.

To solve the y-intercept(s), let x be zero and solve the equation for y.

Example 2 Find the x- and y-intercepts of the graph of $y = x^3 - 4x$.

x-int

$0 = x^3 - 4x$

$= x(x^2 - 4)$

$= x(x+2)(x-2)$

$x = 0, x = \pm 2$

y-int

$y = 0$

Tests for Symmetry
1. The graph of an equation in x and y is symmetric with respect to the y-axis if replacing x by $-x$ yields an equivalent equation. 2. The graph of an equation in x and y is symmetric with respect to the x-axis if replacing y by $-y$ yields an equivalent equation. 3. The graph of an equation in x and y is symmetric with respect to the origin if replacing x by $-x$ and y by $-y$ yields an equivalent equation.

Example 3 Check for symmetry with the equation $y = 2x^3 - x$

y-axis

$y = 2(-x)^3 - (-x)$

$y = -2x^3 + x$

No

x-axis

$-y = 2x^3 - x$

No

origin

$-y = 2(-x)^3 - (-x)$

$-y = -2x^3 + x$

YES

Using Intercepts and Symmetry to sketch a graph

Example 4 Sketch the graph of $x - y^2 = 1$

Start by finding symmetry, then depending on symmetry, find intercepts

x-axis symmetry y-axis symmetry origin symmetry

$x - (-y)^2 = 1$

$x - y^2 = 1$

yes

Now that we know the graph is symmetric with respect to the x-axis, let's find the x-intercept.

$x-(0)^2=1$

$x=1$ $(1,0)$

$x-y^2=1$

$y^2=x-1$

$y=\sqrt{x-1}$

x	0	2	3	4	5
y	X	1	$\sqrt{2}$	$\sqrt{3}$	2

$x=0$ $y=\sqrt{0-1}$

$y=i$

$x=2$ $y=\sqrt{2-1}$

$y=1$ ± 1

$x=3$ $y=\sqrt{3-1}$

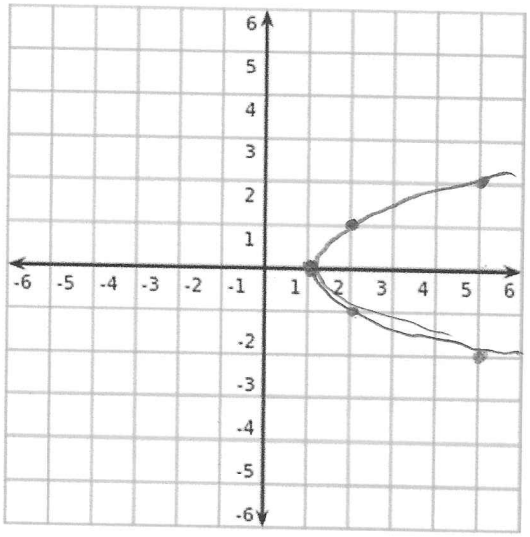
$=\sqrt{2}$

$x=4$ $y=\sqrt{4-1}$

$=\sqrt{3}$

$x=5$ $y=\sqrt{5-1}$

$=2$



In Exercises 19–26, find any intercepts.

19. $y = x^2 + x - 2$
20. $y^2 = x^3 - 4x$
21. $y = x^2\sqrt{25 - x^2}$
22. $y = (x - 1)\sqrt{x^2 + 1}$
23. $y = \dfrac{3(2 - \sqrt{x})}{x}$
24. $y = \dfrac{x^2 + 3x}{(3x + 1)^2}$
25. $x^2y - x^2 + 4y = 0$
26. $y = 2x - \sqrt{x^2 + 1}$

In Exercises 27–38, test for symmetry with respect to each axis and to the origin.

27. $y = x^2 - 2$
28. $y = x^2 - x$
29. $y^2 = x^3 - 4x$
30. $y = x^3 + x$
31. $xy = 4$
32. $xy^2 = -10$
33. $y = 4 - \sqrt{x + 3}$
34. $xy - \sqrt{4 - x^2} = 0$
35. $y = \dfrac{x}{x^2 + 1}$
36. $y = \dfrac{x^2}{x^2 + 1}$
37. $y = |x^3 + x|$
38. $|y| - x = 3$

In Exercises 39–56, sketch the graph of the equation. Identify any intercepts and test for symmetry.

39. $y = -3x + 2$
40. $y = -\frac{1}{2}x + 2$
41. $y = \frac{1}{2}x - 4$
42. $y = \frac{2}{3}x + 1$
43. $y = 1 - x^2$
44. $y = x^2 + 3$
45. $y = (x + 3)^2$
46. $y = 2x^2 + x$
47. $y = x^3 + 2$
48. $y = x^3 - 4x$
49. $y = x\sqrt{x + 2}$
50. $y = \sqrt{9 - x^2}$
51. $x = y^3$
52. $x = y^2 - 4$
53. $y = \dfrac{1}{x}$
54. $y = \dfrac{10}{x^2 + 1}$
55. $y = 6 - |x|$
56. $y = |6 - x|$

HW: 19-21, 27-29, 42, 44, 48