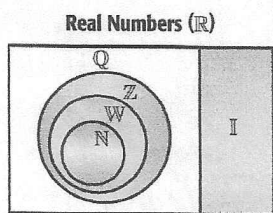


P6 – Functions, Domain and Range



Letter	Set	Examples
Q	rational	$0.125, -\frac{7}{8}, \frac{2}{3} = 0.666\dots$
I	irrational	$\sqrt{3} = 1.73205\dots$
Z	integers	$-5, 17, -23, 8$
W	wholes	$0, 1, 2, 3\dots$
N	naturals	$1, 2, 3, 4\dots$

Set Builder Notation – uses the properties of the numbers in the set to define the set

$$\{x\}$$

$$\{x \mid -3 \leq x \leq 16, x \in \mathbb{Z}\}$$

the set of numbers x such that	x has the given properties $-3 \leq x \leq 16$	and x is an element of the given set of numbers
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Example 1 Describe the set of numbers using set builder notation

a. $\{8, 9, 10, 11, \dots\}$

$$\{x \mid x \geq 8, x \in \mathbb{N}\}$$

b. $x < 7$

$$\{x \mid x < 7, x \in \mathbb{R}\}$$

c. Multiples of 3

$$\{x \mid x = 3n, x \in \mathbb{Z}\}$$

Interval Notation – uses inequalities to describe subsets of real numbers

Bounded	Intervals	Unbounded	Intervals
Inequality	Interval Notation	Inequality	Interval Notation
$a \leq x \leq b$	$[a, b]$	$x \geq a$	$[a, \infty)$
$a < x < b$	(a, b)	$x \leq a$	$(-\infty, a]$
$a \leq x < b$	$[a, b)$	$x > a$	(a, ∞)
$a < x \leq b$	$(a, b]$	$x < a$	$(-\infty, a)$
		$-\infty < x < \infty$	$(-\infty, \infty)$

Example 2 Write each set of numbers using interval notation

a. $-8 < x \leq 16$

$$(-8, 16]$$

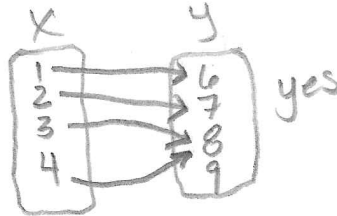
b. $x < 11$

$$(-\infty, 11)$$

Function – a relation that assigns to each element x in set A exactly one element y in set B , or a set of ordered pairs in which no two different pairs have the same x -value

x	y
-2	-4
3	-1
3	4
5	6

Not a function



Vertical Line Test – if you can draw a vertical line anywhere on the graph of a relation and not intersect more than once, then the graph represents a function

Example 3

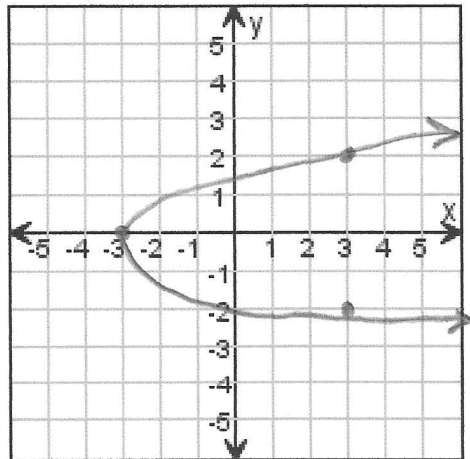
a.

x	y
-8	-5
-5	-4
0	-3
3	-2
6	-3

Yes

b.

No



Functional Notation – the symbol $f(x)$ is read f of x and is interpreted as the value of the function f at x . Because $f(x)$ corresponds to the y -value for a given x -value, we can say that $y = f(x)$

Example 4 If $g(x) = x^2 + 8x - 24$, find each function value

a. $g(6) = (6)^2 + 8(6) - 24$
 $= 36 + 48 - 24 = 60$

b. $g(-4x) = (-4x)^2 + 8(-4x) - 24$
 $= 16x^2 - 32x - 24$

Implied Domain – the set of all real numbers for which the expression used to define the function is real. In general, we must exclude values from the domain that would result in division by zero or taking the root of a negative number.

Example 5 State the domain of the function

a. $f(x) = \frac{2+x}{x^2-7x}$

b. $h(a) = \sqrt{a^2 - 4}$

$x^2 - 7x = 0$
 $x(x - 7) = 0$
 $x = 0 \quad x = 7$
 $D: \{x \mid x \neq 0, x \neq 7, x \in \mathbb{R}\}$

$a^2 - 4 = 0$
 $(a+2)(a-2) = 0$
 $a = \pm 2$
 $D: \{x \mid x \geq 2, x \in \mathbb{R}\}$

Write each set of numbers in set-builder and interval notation, if possible. (Examples 1 and 2) **1–14. See margin.**

1. $x > 50$
2. $x < -13$
3. $x \leq -4$
4. $\{-4, -3, -2, -1, \dots\}$
5. $8 < x < 99$
6. $-31 < x \leq 64$

Determine whether each relation represents y as a function of x . (Example 3)

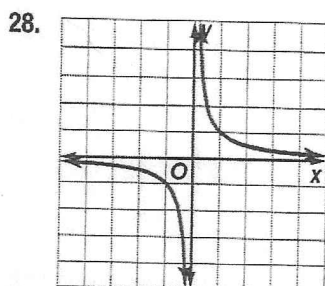
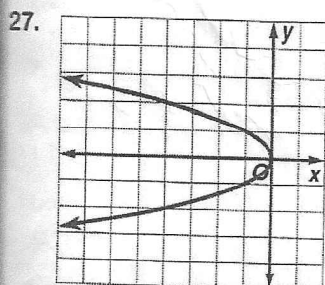
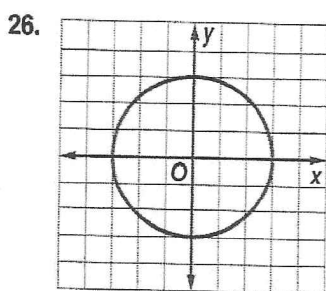
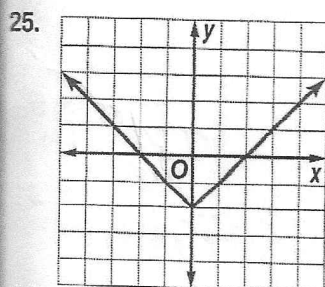
15. The input value x is a bank account number and the output value y is the account balance.
16. The input value x is the year and the output value y is the day of the week.

17.

x	y
-50	2.11
-40	2.14
-30	2.16
-20	2.17
-10	2.17
0	2.18

18.

x	y
0.01	423
0.04	449
0.04	451
0.07	466
0.08	478
0.09	482



Find each function value. (Example 4)

30. $g(x) = 2x^2 + 18x - 14$
31. $h(y) = -3y^3 - 6y + 9$
- a. $g(9)$
- a. $h(4)$
- b. $g(3x)$
- b. $h(-2y)$
- c. $g(1 + 5m)$
- c. $h(5b + 3)$

State the domain of each function.

39. $f(x) = \frac{8x + 12}{x^2 + 5x + 4}$
40. $g(x) = \frac{x + 1}{x^2 - 3x - 40}$
41. $g(a) = \sqrt{1 + a^2}$
42. $h(x) = \sqrt{6 - x^2}$
43. $f(a) = \frac{5a}{\sqrt{4a - 1}}$
44. $g(x) = \frac{3}{\sqrt{x^2 - 16}}$
45. $f(x) = \frac{2}{x} + \frac{4}{x + 1}$
46. $g(x) = \frac{6}{x + 3} + \frac{2}{x - 4}$