P7 - Composite Functions

Functional Operations involved adding, subtracting, multiplying, or dividing two functions.

A composite function uses one function to evaluate another function.

The composition of function *f* with function g is defined by: $[f \circ g](x) = f[g(x)]$

Example 1 Given $f(x) = x^2 + 1$ and $g(x) = x - 4$, find each of the following

a. $[f \circ g](x)$

$= (x-4)^2 + 1$

$= (x^2 - 8x + 16) + 1$

$= x^2 - 8x + 17$

b. $[g \circ f](x)$

$= (x^2 + 1) - 4$

$= x^2 + 1 - 4$

$= x^2 - 3$

c. $[f \circ g](2)$

$= (2)^2 - 8(2) + 17$

$= 4 - 16 + 17$

$= 5$

Many times in Calculus we must be able to decompose a function into two simpler functions. To decompose a function *h*, find two functions with a composition of *h*.

Example 2 Find two functions *f* and g, such that $h(x) = [f \circ g](x)$

a. $h(x) = \sqrt{x^3 - 4} \rightarrow g(x) = x^3 - 4$

$f(x) = \sqrt{x}$

b. $h(x) = 2x^2 + 20x + 50$

2

$= 2(x^2 + 10x + 25)$

$= 2(x+5)(x+5)$

$= 2(x+5)^2 \rightarrow f(x) = 2x^2$

$g(x) = x+5$

HW: pg. 61; 16, 18, 22, 24, 30, 32, 36

Exercises

= Step-by-Step Solutions begin on page R29.

Find $(f+g)(x)$, $(f-g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$. State the domain of each new function. (Example 1)

1. $f(x) = x^2 + 4$, $g(x) = \sqrt{x}$
2. $f(x) = 8 - x^3$, $g(x) = x - 3$
3. $f(x) = x^2 + 5x + 6$, $g(x) = x + 2$
4. $f(x) = x - 9$, $g(x) = x + 5$
5. $f(x) = x^2 + x$, $g(x) = 9x$
6. $f(x) = x - 7$, $g(x) = x + 7$
7. $f(x) = \frac{6}{x}$, $g(x) = x^3 + x$
8. $f(x) = \frac{x}{4}$, $g(x) = \frac{3}{x}$
9. $f(x) = \frac{1}{\sqrt{x}}$, $g(x) = 4\sqrt{x}$
10. $f(x) = \frac{3}{x}$, $g(x) = x^4$
11. $f(x) = \sqrt{x+8}$, $g(x) = \sqrt{x+5} - 3$
12. $f(x) = \sqrt{x+6}$, $g(x) = \sqrt{x-4}$

13. **BUDGETING** Suppose a budget in dollars for one person for one month is approximated by $f(x) = 25x + 350$ and $g(x) = 15x + 200$, where f is the cost of rent and groceries, g is the cost of gas and all other expenses, and $x = 1$ represents the total cost at the end of the first week. (Example 1)
    a. Find $(f+g)(x)$ and the relevant domain.
    b. What does $(f+g)(x)$ represent?
    c. Find $(f+g)(4)$. What does this value represent?

14. **PHYSICS** Two different forces act on an object being pushed across a floor: the force of the person pushing the object and the force of friction. If W is work in joules, F is force in newtons, and d is displacement of the object in meters, $W_p(d) = F_p d$ describes the work of the person and $W_f(d) = F_f d$ describes the work done by friction. The increase in kinetic energy of the object is the difference between the work done by the person W_p and the work done by friction W_f. (Example 1)
    a. Find $(W_p - W_f)(d)$.
    b. Determine the net work expended when a person pushes a box 50 meters with a force of 95 newtons and friction exerts a force of 55 newtons.

For each pair of functions, find $[f \circ g](x)$, $[g \circ f](x)$, and $[f \circ g](6)$. (Example 2)

15. $f(x) = 2x - 3$, $g(x) = 4x - 8$
16. $f(x) = -2x^2 - 5x + 1$, $g(x) = -5x + 6$
17. $f(x) = 8 - x^2$, $g(x) = x^2 + x + 1$
18. $f(x) = x^2 - 16$, $g(x) = x^2 + 7x + 11$
19. $f(x) = 3 - x^2$, $g(x) = x^3 + 1$
20. $f(x) = 2 + x^4$, $g(x) = -x^2$

Find $f \circ g$. (Example 3)

21. $f(x) = \frac{1}{x+1}$, $g(x) = x^2 - 4$
22. $f(x) = \frac{2}{x-3}$, $g(x) = x^2 + 6$
23. $f(x) = \sqrt{x+4}$, $g(x) = x^2 - 4$
24. $f(x) = x^2 - 9$, $g(x) = \sqrt{x+3}$
25. $f(x) = \frac{5}{x}$, $g(x) = \sqrt{6-x}$
26. $f(x) = -\frac{4}{x}$, $g(x) = \sqrt{x+8}$
27. $f(x) = \sqrt{x+5}$, $g(x) = x^2 + 4x - 1$
28. $f(x) = \sqrt{x-2}$, $g(x) = x^2 + 8$

29. **RELATIVITY** In the theory of relativity, $m(v) = \frac{100}{\sqrt{1 - \frac{v^2}{c^2}}}$, where c is the speed of light, 300 million meters per second, and m is the mass of a 100-kilogram object at speed v in meters per second. (Example 4)
    a. Are there any restrictions on the domain of the function? Explain their meaning.
    b. Find $m(10)$, $m(10{,}000)$, and $m(1{,}000{,}000)$.
    c. Describe the behavior of $m(v)$ as v approaches c.
    d. Decompose the function into two separate functions.

Find two functions f and g such that $h(x) = [f \circ g](x)$. Neither function may be the identity function $f(x) = x$. (Example 4)

30. $h(x) = \sqrt{4x+2} + 7$
31. $h(x) = \frac{6}{x+5} - 8$
32. $h(x) = |4x + 8| - 9$
33. $h(x) = [\![-3(x-9)]\!]$
34. $h(x) = \sqrt{\frac{5-x}{x+2}}$
35. $h(x) = (\sqrt{x} + 4)^3$
36. $h(x) = \frac{6}{(x+2)^2}$
37. $h(x) = \frac{8}{(x-5)^2}$
38. $h(x) = \frac{\sqrt{4+x}}{x-2}$
39. $h(x) = \frac{x+5}{\sqrt{x}-1}$

40. **QUANTUM MECHANICS** The wavelength λ of a particle with mass m kilograms moving at v meters per second is represented by $\lambda = \frac{h}{mv}$, where h is a constant equal to $6.626 \cdot 10^{-34}$.
    a. Find a function to represent the wavelength of a 25-kilogram object as a function of its speed.
    b. Are there any restrictions on the domain of the function? Explain their meaning.
    c. If the object is traveling 8 meters per second, find the wavelength in terms of h.
    d. Decompose the function into two separate functions.