

P8

~~11th~~ nth Roots and Real Exponents

Simplifying Radicals

if $b^n = a$, then b is an n th root of a $a=9$ $b=3$ $n=2$

$$3^2 = 9 \qquad 3 = \sqrt{9}$$

Real n th Roots of Real Numbers

Suppose n is an integer greater than 1, and a is a real number

a	n is even	n is odd
$a > 0$	1 positive and 1 negative real root: $\pm \sqrt[n]{a}$	1 positive and 0 negative real root: $\sqrt[n]{a}$
$a < 0$	0 real roots	0 positive and 1 negative real root: $\sqrt[n]{a}$
$a = 0$	1 real root: $\sqrt[n]{0} = 0$	1 real root: $\sqrt[n]{0} = 0$

Example 1 Evaluate

a. $-\sqrt{49} = -7$

b. $\sqrt[3]{\frac{27}{64}} = \frac{\sqrt[3]{27}}{\sqrt[3]{64}} = \frac{3}{4}$

c. $\sqrt[4]{-121} = \text{no R roots}$

Product Property $\sqrt[n]{a} * \sqrt[n]{b} = \sqrt[n]{ab}$

Quotient Property $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$

Example 2 Simplify

a. $\sqrt[6]{n^{18}} = \sqrt[6]{(n^3)^6} = n^3$

b. $\sqrt{63y^3} = \sqrt{9y^2} \sqrt{7y}$
 $= 3y\sqrt{7y}$

c. $\sqrt[4]{81(a+1)^{12}} = \sqrt[4]{81} \sqrt[4]{[(a+1)^3]^4}$
 $= 3(a+1)^3$

c. $\sqrt[5]{-p^{10}q^7} = \sqrt[5]{(-p^2q)^5} \sqrt[5]{q^2}$
 $= -p^2q\sqrt[5]{q^2}$

Rational Exponents

If b is a real number, variable, or algebraic expression and m and n are positive integers greater than 1, then:

$b^{\frac{1}{n}} = \sqrt[n]{b}$, if the principal n th root of b exists, and

$b^{\frac{m}{n}} = (\sqrt[n]{b})^m$ or $\sqrt[n]{b^m}$, if $\frac{m}{n}$ is in reduced form

Example 3 Simplify

$$\text{a. } \frac{10\sqrt[3]{32}}{6\sqrt{2}} = \frac{2^{\frac{5}{10}}}{2^{\frac{1}{6}}} = 2^{\frac{5}{10} - \frac{1}{6}} = 2^{\frac{15}{30} - \frac{5}{30}} = 2^{\frac{10}{30}} = 2^{\frac{1}{3}} = \sqrt[3]{2}$$

$$\text{b. } \sqrt[4]{x^{21}y^{15}} = x^{\frac{21}{4}} \cdot y^{\frac{15}{4}} = x^{\frac{20}{4}} x^{\frac{1}{4}} y^{\frac{12}{4}} y^{\frac{3}{4}} = x^5 y^3 \sqrt[4]{xy^3}$$

$$\text{c. } \frac{16^{\frac{3}{4}}}{16^{\frac{1}{4}}} = 16^{\frac{3}{4} - \frac{1}{4}} = 16^{\frac{2}{4}} = 16^{\frac{1}{2}} = \sqrt{16} = 4$$

Evaluate. (Example 1)

1. $-\sqrt{169}$

2. $\sqrt{-100}$

3. $\sqrt[3]{\frac{216}{125}}$

4. $\sqrt[3]{-\frac{64}{343}}$

5. $\sqrt[4]{-81}$

6. $\sqrt[4]{625}$

7. $\sqrt[5]{243}$

8. $\sqrt[5]{-1024}$

Simplify. (Example 2)

9. $\sqrt[3]{-27x^9}$

10. $\sqrt[4]{16a^{20}}$

11. $\sqrt[8]{8y^{16}}$

12. $\sqrt[3]{54x^{17}}$

13. $\sqrt{20x^{16}}$

14. $\sqrt{121(z-2)^{14}}$

15. $\sqrt[4]{a^{12}b^9}$

16. $\sqrt[7]{-q^{13}r^{16}}$

Simplify. (Example 3)

17. $\frac{b^{\frac{5}{4}} \cdot b^{\frac{3}{4}}}{b^{\frac{1}{4}}}$

18. $(2x^{\frac{1}{4}}y^{\frac{1}{3}})(3x^{\frac{1}{4}}y^{\frac{2}{3}})$

19. $\sqrt[6]{640a^3}$

20. $\sqrt[6]{128b^4}$

21. $\frac{\sqrt[3]{16}}{\sqrt[5]{4}}$

22. $\frac{\sqrt[4]{27}}{\sqrt[3]{81}}$