Chapman Pre Cal

P9 - Exponential Functions

$f(x) = b^x$ is an exponential function where $b > 0, b \neq 1$.

Example 1 Graphing Exponential Functions

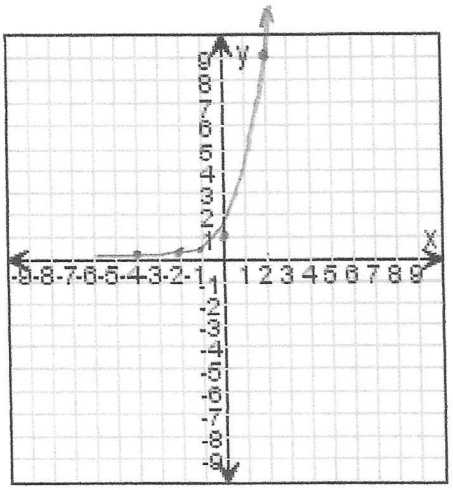
Graph $f(x) = 3^x$

x	-4	-2	-1	0	2	4	6
$f(x)$	.01	.11	.33	1	9	81	729

$3^{-4} = \frac{1}{3^4} = \frac{1}{81} = .01$

$3^{-2} = \frac{1}{9} = .11$

$3^{-1} = \frac{1}{3} = .3$



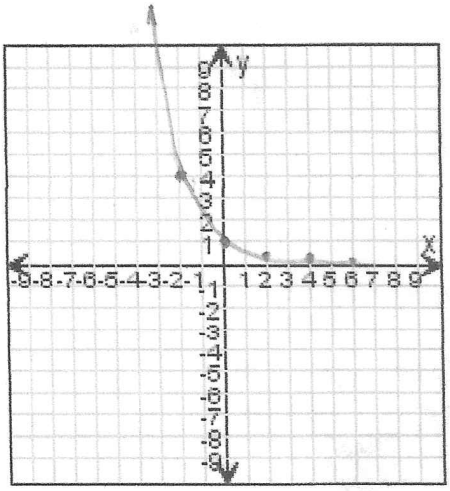
Example 2 Graph $f(x) = 2^{-x}$

x	-6	-4	-2	0	2	4	6
$f(x)$	64	16	4	1	.25	.0625	.02

$2^{-(-6)} = 2^6 = 64$

$2^{-2} = \frac{1}{2^2} = \frac{1}{4} = .25$

$2^{-4} = \frac{1}{2^4} = \frac{1}{16}$



Compound Interest → $A = P\left(1 + \frac{r}{n}\right)^{nt}$

where: A = account balance
P = principal (initial investment)
r = interest rate
n = number of times P is compounded each year.
t = number of years

Example 3 Krysti invests \$300 in an account with a 6% interest rate, making no other deposits or withdrawals. What will Krysti's account balance be after 20 years if the interest is compounded:

a. semiannually? $n = 2$

$A = 300\left(1 + \frac{.06}{2}\right)^{2(20)}$

$= 300(1.03)^{40}$

$= \$978.61$

b. monthly? $n = 12$

$A = 300\left(1 + \frac{.06}{12}\right)^{12(20)}$

$= 300(1.005)^{240}$

$= \$993.06$

c. daily? $n = 365$

$A = 300\left(1 + \frac{.06}{365}\right)^{365(20)}$

$= 300\left(1 + \frac{.06}{365}\right)^{7300}$ $= \$995.94$

Continuous Compound Interest $A = Pe^{rt}$

Example 4 Suppose Krysti finds an account that will allow her to invest her \$300 at a 6% interest rate compounded continuously. If there are no other deposits or withdrawals, what will Krysti's account balance be after 20 years? $A = 300e^{.06(20)}$

$= \$996.04$

Normal Exponential Growth and Decay – If an initial quantity N_0 grows or decays at an exponential rate r or k (as a decimal), then the final amount N after a time t is given by the following formulas

Exponential Growth → $N = N_0(1 + r)^t$

$r > 0$, growth
$r < 0$, decay

Continuous Exponential Growth → $N = N_0e^{kt}$

$k > 0$, continuous growth
$k < 0$, continuous decay

P9

3.1 Homework

Graph the exponential functions

2. $r(x) = 5^x$

4. $k(x) = 6^x$

6. $p(x) = 0.1^{-x}$

8. $g(x) = \left(\frac{1}{3}\right)^x$

10. $d(x) = 5^{-x} + 2$

Answer the following

26. FINANCIAL LITERACY Katrina invests $1200 in a certificate of deposit (CD). The table shows the interest rates offered by the bank on 3- and 5-year CDs. (Examples 4 and 5)

CD Offers		
Years	3	5
Interest	3.45%	4.75%
Compounded	continuously	monthly

a. How much would her investment be worth with each option?

b. How much would her investment be worth if the 5-year CD was compounded continuously?

POPULATION Copy and complete the table to find the population N of an endangered species after a time t given its initial population N_0 and annual rate r or continuous rate k of increase or decline. (Example 6)

t	5	10	15	20	50
N					

27. $N_0 = 15{,}831, r = -4.2\%$ **28.** $N_0 = 23{,}112, r = 0.8\%$

29. $N_0 = 17{,}692, k = 2.02\%$ **30.** $N_0 = 9689, k = -3.7\%$

31. WATER Worldwide water usage in 1950 was about 294.2 million gallons. If water usage has grown at the described rate, estimate the amount of water used in 2000 and predict the amount in 2050. (Example 6)

a. 3% annually **b.** 3.05% continuously