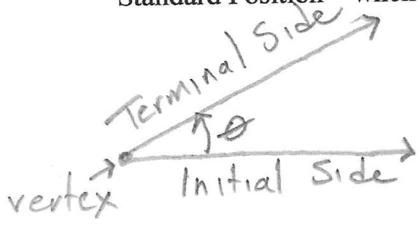
T3 - Degrees and Radians

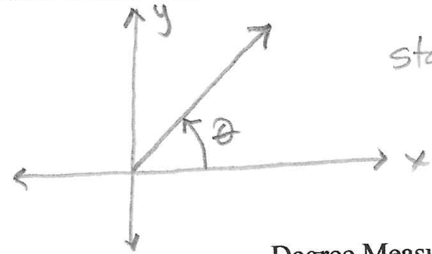
Vertex – point where two noncollinear rays share an endpoint

Initial Side – starting position of a ray

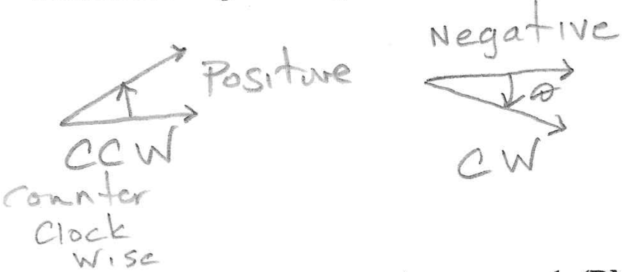
Terminal Side – where the rotation of a ray stops to form an angle θ

Standard Position – when an angle's initial side is the x-axis and the vertex of the angle is the origin

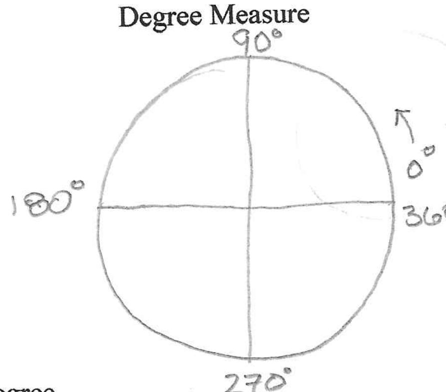




Positive and Negative Angles



Degree Measure



Decimal Degree and Degree-minutes-seconds (DMS)

Degree °
Minute ′
Second ″

Example 1 Convert between DMS and decimal degree

a) 56.735°

$56° + 0.735°\left(\frac{60'}{1°}\right) = 56°\,44.1'$

$56°\,44' + 0.1'\left(\frac{60''}{1'}\right) = \boxed{56°\,44'\,6''}$

b) 213.875°

$213° + 0.875°\left(\frac{60'}{1°}\right) = 213°\,52.5'$

$213°\,52' + 0.5'\left(\frac{60''}{1'}\right) = \boxed{213°\,52'\,30''}$

c) 32° 5' 28"

$32° + 5'\left(\frac{1°}{60'}\right) + \left(\frac{1°}{3600''}\right)$

$= 32° + 0.083° + 0.008°$

$= 32.091°$

d) 89° 56' 7"

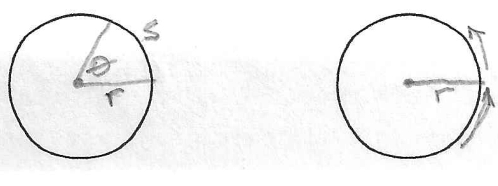
$89° + 56'\left(\frac{1°}{60'}\right) + 7''\left(\frac{1°}{3600''}\right)$

$= 89° + 0.93° + 0.002°$

$= 89.932°$

Radian Measure (rad) – The measure θ in radians of a central angel of a circle is equal to the ratio of the length of the intercepted arc s to the radius r of the circle.

$\theta = \frac{s}{r}$ $\qquad\qquad$ $\theta = 1$ when $r = s$



when s is a complete revolution

$s = C = 2\pi r$ there fore

$\theta = \frac{s}{r} = \frac{2\pi r}{r} = 2\pi$ radians

Measuring Radians

$\theta = \frac{s}{r} = \frac{1}{2} \cdot \frac{2\pi r}{r}$ $\quad$ $\theta = \frac{s}{r} = \frac{1}{4} \cdot \frac{2\pi r}{r}$ $\quad$ $\theta = \frac{s}{r} = \frac{3}{4} \cdot \frac{2\pi r}{r}$ $\quad$ $\theta = \frac{s}{r} = \frac{1}{8} \cdot \frac{2\pi r}{r}$

$\theta = \pi$ radians $\quad$ $\theta = \frac{\pi}{2}$ radians $\quad$ $\theta = \frac{6\pi}{4} = \frac{3\pi}{2}$ radians $\quad$ $\theta = \frac{\pi}{4}$ radians

Degree/Radian Conversion

Degrees to Radians	$D°\left(\frac{\pi}{180°}\right) = rad$	multiply by $\frac{\pi}{180°}$
Radians to Degrees	$rad\left(\frac{180°}{\pi}\right) = D°$	multiply by $\frac{180°}{\pi}$

Example 2 Convert

a) 120°

$120°\left(\frac{\pi}{180}\right) = \frac{120\pi}{180} = \frac{2\pi}{3}$ rad

b) -45° $\quad -45°\left(\frac{\pi}{180}\right) = \frac{-45\pi}{180}$

$= \frac{-\pi}{4}$

c) $\frac{5\pi}{6}\left(\frac{180}{\pi}\right) = \frac{900\pi}{6\pi} = 150°$

d) $-\frac{3\pi}{2}\left(\frac{180}{\pi}\right) = -\frac{540\pi}{2\pi} = -270°$

Coterminal Angles – Two different angles that have the same initial and terminal sides

Degrees	Radians
$\alpha + 360n°$	$\alpha + 2n\pi$

α = alpha

β = beta

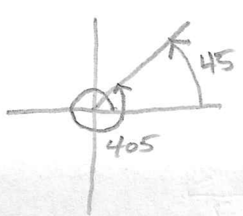
Example 3 Identify all angles that are coterminal with the given angle. Then find and draw one positive and one negative angle coterminal with the given angle.

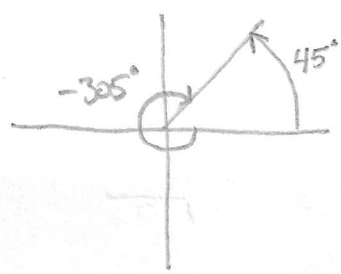
a) 45°

Positive Coterminal $\quad$ Negative

$\alpha + 360(1)° = \beta$ $\quad$ $\alpha + 360(-1)°$

$45 + 360 = 405°$ $\quad$ $45° - 360 = -315°$





b) $-\frac{\pi}{3}$ $\quad$ + $\quad$ −

$\alpha + 2(1)\pi$ $\quad$ $\alpha + 2\pi(-1)$

$= -\frac{\pi}{3} + 2\pi$ $\quad$ $= -\frac{\pi}{3} - 2\pi$

$= -\frac{\pi}{3} + \frac{6\pi}{3}$ $\quad$ $= -\frac{\pi}{3} - \frac{6\pi}{3}$

$= \frac{5\pi}{3}$ rad $\quad$ $= -\frac{7\pi}{3}$

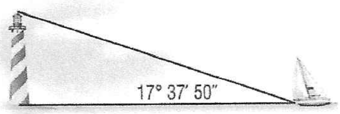
HW: (1-33)

T3 – Degrees and Radians – HW : (1-33)

Write each decimal degree measure in DMS form and each DMS measure in decimal degree form to the nearest thousandth. (Example 1)

1. $11.773°$
2. $58.244°$
3. $141.549°$
4. $273.396°$
5. $87° 53' 10''$
6. $126° 6' 34''$
7. $45° 21' 25''$
8. $301° 42' 8''$

9. **NAVIGATION** A sailing enthusiast uses a sextant, an instrument that can measure the angle between two objects with a precision to the nearest 10 seconds, to measure the angle between his sailboat and a lighthouse. If his reading is $17° 37' 50''$, what is the measure in decimal degree form to the nearest hundredth? (Example 1)



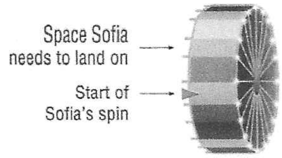
Write each degree measure in radians as a multiple of π and each radian measure in degrees. (Example 2)

10. $30°$
11. $225°$
12. $-165°$
13. $-45°$
14. $\frac{2\pi}{3}$
15. $\frac{5\pi}{2}$
16. $-\frac{\pi}{4}$
17. $-\frac{7\pi}{6}$

Identify all angles that are coterminal with the given angle. Then find and draw one positive and one negative angle coterminal with the given angle. (Example 3)

18. $120°$
19. $-75°$
20. $225°$
21. $-150°$
22. $\frac{\pi}{3}$
23. $-\frac{3\pi}{4}$
24. $-\frac{\pi}{12}$
25. $\frac{3\pi}{2}$

26. **GAME SHOW** Sofia is spinning a wheel on a game show. There are 20 values in equal-sized spaces around the circumference of the wheel. The value that Sofia needs to win is two spaces above the space where she starts her spin, and the wheel must make at least one full rotation for the spin to count. Describe a spin rotation in degrees that will give Sofia a winning result. (Example 3)



Find the length of the intercepted arc with the given central angle measure in a circle with the given radius. Round to the nearest tenth. (Example 4)

27. $\frac{\pi}{6}$, $r = 2.5$ m
28. $\frac{2\pi}{3}$, $r = 3$ in.
29. $\frac{5\pi}{12}$, $r = 4$ yd
30. $105°$, $r = 18.2$ cm
31. $45°$, $r = 5$ mi
32. $150°$, $r = 79$ mm

33. **AMUSEMENT PARK** A carousel at an amusement park rotates $3024°$ per ride. (Example 4)
    a. How far would a rider seated 13 feet from the center of the carousel travel during the ride?
    b. How much farther would a second rider seated 18 feet from the center of the carousel travel during the ride than the rider in part a?

Find the rotation in revolutions per minute given the angular speed and the radius given the linear speed and the rate of rotation. (Example 5)

34. $\omega = \frac{2}{3}\pi \frac{\text{rad}}{\text{s}}$
35. $\omega = 135\pi \frac{\text{rad}}{\text{h}}$
36. $\omega = 104\pi \frac{\text{rad}}{\text{min}}$
37. $v = 82.3 \frac{\text{m}}{\text{s}}, 131 \frac{\text{rev}}{\text{min}}$
38. $v = 144.2 \frac{\text{ft}}{\text{min}}, 10.9 \frac{\text{rev}}{\text{min}}$
39. $v = 553 \frac{\text{in.}}{\text{h}}, 0.09 \frac{\text{rev}}{\text{min}}$

40. **MANUFACTURING** A company manufactures several circular saws with the blade diameters and motor speeds shown below. (Example 5)

Blade Diameter (in.)	Motor Speed (rps)
3	2800
5	5500
$5\frac{1}{2}$	4500
$6\frac{1}{8}$	5500
$7\frac{1}{4}$	5000

    a. Determine the angular and linear speeds of the blades in each saw. Round to the nearest tenth.
    b. How much faster is the linear speed of the $6\frac{1}{8}$-inch saw compared to the 3-inch saw?

41. **CARS** On a stretch of interstate, a vehicle's tires range between 646 and 840 revolutions per minute. The diameter of each tire is 26 inches. (Example 5)
    a. Find the range of values for the angular speeds of the tires in radians per minute.
    b. Find the range of values for the linear speeds of the tires in miles per hour.