

T4 - Trig Values on the Unit Circle

Key Concept

Trigonometric Functions of Any Angle

Let θ be any angle in standard position and point $P(x, y)$ be a point on the terminal side of θ . Let r represent the nonzero distance from P to the origin. That is, let $r = \sqrt{x^2 + y^2} \neq 0$. Then the trigonometric functions of θ are as follows.

$$\sin \theta = \frac{y}{r}$$

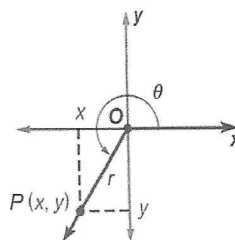
$$\csc \theta = \frac{r}{y}, y \neq 0$$

$$\cos \theta = \frac{x}{r}$$

$$\sec \theta = \frac{r}{x}, x \neq 0$$

$$\tan \theta = \frac{y}{x}, x \neq 0$$

$$\cot \theta = \frac{x}{y}, y \neq 0$$



Example 1 Evaluating Trig Functions at a given point

Let $(8, -6)$ be a point on the terminal side of angle θ . Find the values of the six trig functions for θ

$$x = 8 \quad y = -6 \quad r = \sqrt{x^2 + y^2} = \sqrt{8^2 + (-6)^2} = \sqrt{64 + 36} = \sqrt{100} = 10$$

$$\sin \theta = \frac{-6}{10} = -\frac{3}{5} \quad \csc \theta = -\frac{5}{3}$$

$$\cos \theta = \frac{8}{10} = \frac{4}{5} \quad \sec \theta = \frac{5}{4}$$

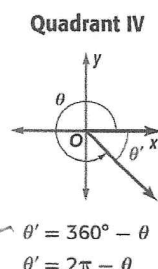
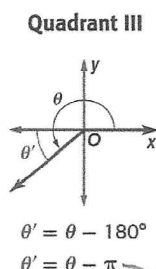
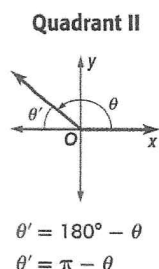
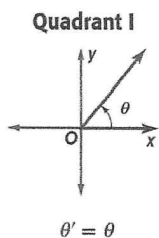
$$\tan \theta = \frac{-6}{8} = -\frac{3}{4} \quad \cot \theta = -\frac{4}{3}$$

Reference Angles (ALWAYS POSITIVE)

Key Concept

Reference Angle Rules

If θ is an angle in standard position, its reference angle θ' is the acute angle formed by the terminal side of θ and the x -axis. The reference angle θ' for any angle θ , $0^\circ < \theta < 360^\circ$ or $0 < \theta < 2\pi$, is defined as follows.



Example 3 Find reference angles

a) 300° Q4

$$\theta' = 360^\circ - 300$$

$$\theta' = 60^\circ$$

b) $\frac{4\pi}{3}$ Q3

$$\theta' = \theta - \pi$$

$$\theta' = \frac{4\pi}{3} - \frac{3\pi}{3}$$

$$\theta' = \frac{\pi}{3}$$

T4 – HW: 1-8, 17-24

The given point lies on the terminal side of an angle θ in standard position. Find the values of the six trigonometric functions of θ . (Example 1)

- | | |
|-------------|-------------|
| 1. (3, 4) | 2. (-6, 6) |
| 3. (-4, -3) | 4. (2, 0) |
| 5. (1, -8) | 6. (5, -3) |
| 7. (-8, 15) | 8. (-1, -2) |

Find the exact value of each trigonometric function, if defined. If not defined, write *undefined*. (Example 2)

- | | |
|-------------------------|---------------------------------------|
| 9. $\sin \frac{\pi}{2}$ | 10. $\tan 2\pi$ |
| 11. $\cot(-180^\circ)$ | 12. $\csc 270^\circ$ |
| 13. $\cos(-270^\circ)$ | 14. $\sec 180^\circ$ |
| 15. $\tan \pi$ | 16. $\sec\left(-\frac{\pi}{2}\right)$ |

Sketch each angle. Then find its reference angle. (Example 3)

- | | |
|-----------------------|-----------------------|
| 17. 135° | 18. 210° |
| 19. $\frac{7\pi}{12}$ | 20. $\frac{11\pi}{3}$ |
| 21. -405° | 22. -75° |
| 23. $\frac{5\pi}{6}$ | 24. $\frac{13\pi}{6}$ |

T5 – HW: 25-40

Find the exact value of each expression. (Example 4)

- | | |
|----------------------------|---------------------------|
| 25. $\cos \frac{4\pi}{3}$ | 26. $\tan \frac{7\pi}{6}$ |
| 27. $\sin \frac{3\pi}{4}$ | 28. $\cot(-45^\circ)$ |
| 29. $\csc 390^\circ$ | 30. $\sec(-150^\circ)$ |
| 31. $\tan \frac{11\pi}{6}$ | 32. $\sin 300^\circ$ |

Find the exact values of the five remaining trigonometric functions of θ . (Example 5)

33. $\tan \theta = 2$, where $\sin \theta > 0$ and $\cos \theta > 0$
34. $\csc \theta = 2$, where $\sin \theta > 0$ and $\cos \theta < 0$
35. $\sin \theta = -\frac{1}{5}$, where $\cos \theta > 0$
36. $\cos \theta = -\frac{12}{13}$, where $\sin \theta < 0$
37. $\sec \theta = \sqrt{3}$, where $\sin \theta < 0$ and $\cos \theta > 0$
38. $\cot \theta = 1$, where $\sin \theta < 0$ and $\cos \theta < 0$
39. $\tan \theta = -1$, where $\sin \theta < 0$
40. $\cos \theta = -\frac{1}{2}$, where $\sin \theta > 0$