

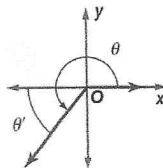
T5 - Trig Values on the Unit Circle

Evaluating Trig Functions at any angle

Key Concept

Evaluating Trigonometric Functions of Any Angle

- Step 1 Find the reference angle θ' .
- Step 2 Find the value of the trigonometric function for θ' .
- Step 3 Using the quadrant in which the terminal side of θ lies, determine the sign of the trigonometric function value of θ .



Quadrant II	Quadrant I
sin θ : +	sin θ : +
cos θ : -	cos θ : +
tan θ : -	tan θ : +
Quadrant III	Quadrant IV
sin θ : -	sin θ : -
cos θ : -	cos θ : +
tan θ : +	tan θ : -

θ	30° or $\frac{\pi}{6}$	45° or $\frac{\pi}{4}$	60° or $\frac{\pi}{3}$
sin θ	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
cos θ	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
tan θ	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

Example 4 Find the exact value of each expression

a) $\cos 120^\circ$

$\theta' = 60^\circ$

$\cos 60^\circ = \frac{1}{2}$

$\cos 120^\circ = -\frac{1}{2}$

b) $\tan \frac{7\pi}{6}$

$\theta = \pi$
 $\theta' = \frac{7\pi}{6} - \frac{6\pi}{6}$
 $\theta' = \frac{\pi}{6}$

$\tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}$

$\tan \frac{7\pi}{6} = -\frac{1}{\sqrt{3}}$

c) $\csc \frac{15\pi}{4}$

$\theta' = 2\pi - \theta$

$\theta' = \frac{16\pi}{4} - \frac{15\pi}{4} = \frac{\pi}{4}$

$\sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$\csc \frac{\pi}{4} = \frac{2}{\sqrt{2}} = \sqrt{2}$

Using one trig value to find others $\tan \theta$ is + $\sin \theta$ is - Q3

Example 5 Let $\tan \theta = \frac{5}{12}$, where $\sin \theta < 0$. Find the exact values of the five remaining trig functions of θ .

$$\tan \theta = \frac{y}{x} = \frac{5}{12} \quad x = +12 \quad y = -5 \quad r = 13$$

$$r = \sqrt{x^2 + y^2} = \sqrt{12^2 + (-5)^2} = \sqrt{144 + 25} = \sqrt{169} = 13$$

$$\sin \theta = \frac{-5}{13}$$

$$\csc \theta = \frac{13}{-5}$$

$$\cos \theta = \frac{-12}{13}$$

$$\sec \theta = \frac{13}{-12}$$

$$\tan \theta = \frac{5}{12}$$

$$\cot \theta = \frac{12}{5}$$

Finding trig values using the unit circle

Key Concept

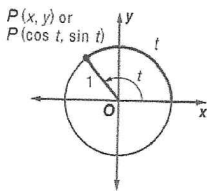
Trigonometric Functions on the Unit Circle

Let t be any real number on a number line and let $P(x, y)$ be the point on t when the number line is wrapped onto the unit circle. Then the trigonometric functions of t are as follows.

$$\sin t = y \quad \cos t = x \quad \tan t = \frac{y}{x}, x \neq 0$$

$$\csc t = \frac{1}{y}, y \neq 0 \quad \sec t = \frac{1}{x}, x \neq 0 \quad \cot t = \frac{x}{y}, y \neq 0$$

Therefore, the coordinates of P corresponding to the angle t can be written as $P(\cos t, \sin t)$.



Example 7 Find the exact value of each expression. If undefined, write UND

a) $\sin \frac{\pi}{3}$

$$\frac{\sqrt{3}}{2}$$

b) $\cos 135^\circ$

$$-\frac{\sqrt{2}}{2}$$

c) $\tan 270^\circ$

$$\tan 270^\circ = \frac{-1}{0} = \text{UND}$$

d) $\csc \frac{11\pi}{6}$

$$\sin \frac{11\pi}{6} = -\frac{1}{2}$$

$$\csc \frac{11\pi}{6} = -2$$

T4 – HW: 1-8, 17-24

The given point lies on the terminal side of an angle θ in standard position. Find the values of the six trigonometric functions of θ . (Example 1)

- | | |
|-------------|-------------|
| 1. (3, 4) | 2. (-6, 6) |
| 3. (-4, -3) | 4. (2, 0) |
| 5. (1, -8) | 6. (5, -3) |
| 7. (-8, 15) | 8. (-1, -2) |

Find the exact value of each trigonometric function, if defined. If not defined, write *undefined*. (Example 2)

- | | |
|-------------------------|---------------------------------------|
| 9. $\sin \frac{\pi}{2}$ | 10. $\tan 2\pi$ |
| 11. $\cot(-180^\circ)$ | 12. $\csc 270^\circ$ |
| 13. $\cos(-270^\circ)$ | 14. $\sec 180^\circ$ |
| 15. $\tan \pi$ | 16. $\sec\left(-\frac{\pi}{2}\right)$ |

Sketch each angle. Then find its reference angle. (Example 3)

- | | |
|-----------------------|-----------------------|
| 17. 135° | 18. 210° |
| 19. $\frac{7\pi}{12}$ | 20. $\frac{11\pi}{3}$ |
| 21. -405° | 22. -75° |
| 23. $\frac{5\pi}{6}$ | 24. $\frac{13\pi}{6}$ |

T5– HW: 25-40

Find the exact value of each expression. (Example 4)

- | | |
|----------------------------|---------------------------|
| 25. $\cos \frac{4\pi}{3}$ | 26. $\tan \frac{7\pi}{6}$ |
| 27. $\sin \frac{3\pi}{4}$ | 28. $\cot(-45^\circ)$ |
| 29. $\csc 390^\circ$ | 30. $\sec(-150^\circ)$ |
| 31. $\tan \frac{11\pi}{6}$ | 32. $\sin 300^\circ$ |

Find the exact values of the five remaining trigonometric functions of θ . (Example 5)

33. $\tan \theta = 2$, where $\sin \theta > 0$ and $\cos \theta > 0$
34. $\csc \theta = 2$, where $\sin \theta > 0$ and $\cos \theta < 0$
35. $\sin \theta = -\frac{1}{5}$, where $\cos \theta > 0$
36. $\cos \theta = -\frac{12}{13}$, where $\sin \theta < 0$
37. $\sec \theta = \sqrt{3}$, where $\sin \theta < 0$ and $\cos \theta > 0$
38. $\cot \theta = 1$, where $\sin \theta < 0$ and $\cos \theta < 0$
39. $\tan \theta = -1$, where $\sin \theta < 0$
40. $\cos \theta = -\frac{1}{2}$, where $\sin \theta > 0$