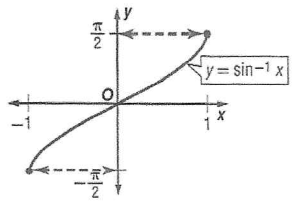
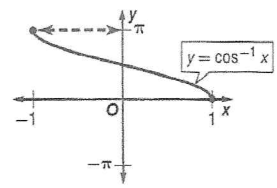
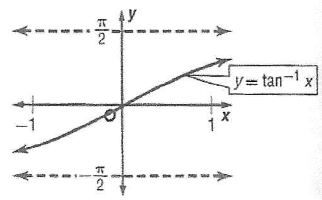
# T6 - Inverse Trig Functions

**Key Concept** **Inverse Trigonometric Functions**

| Inverse Sine of $x$ | Inverse Cosine of $x$ | Inverse Tangent of $x$ |
|---|---|---|
| **Words** The angle (or arc) between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ with a sine of $x$. | **Words** The angle (or arc) between 0 and $\pi$ with a cosine of $x$. | **Words** The angle (or arc) between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ with a tangent of $x$. |
| **Symbols** $y = \sin^{-1} x$ if and only if $\sin y = x$, for $-1 \le x \le 1$ and $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$. | **Symbols** $y = \cos^{-1} x$ if and only if $\cos y = x$, for $-1 \le x \le 1$ and $0 \le y \le \pi$. | **Symbols** $y = \tan^{-1} x$ if and only if $\tan y = x$, for $-\infty < x < \infty$ and $-\frac{\pi}{2} < y < \frac{\pi}{2}$. |
| **Domain:** $[-1, 1]$ | **Domain:** $[-1, 1]$ | **Domain:** $(-\infty, \infty)$ |
| **Range:** $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ | **Range:** $[0, \pi]$ | **Range:** $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ |




Let's use $t$ for $\theta$

Remember that if $\sin t = y$, then $\sin^{-1} y = t$, OR we also say that $\arcsin y = t$

So $\arcsin t = \sin^{-1} t$

$\Rightarrow \arccos t = \cos^{-1} t$ and $\arctan t = \tan^{-1} t$

To sum up, the $\arcsin y = t$ is found over the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

The $\arccos x = t$ is found over the interval $[0, \ \pi]$, and

The $\arctan \frac{y}{x} = t$ is found over the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Example 1 Evaluate inverse sine functions

a) $\sin^{-1} \frac{1}{2}$

$\frac{\pi}{6}$ rad

b) $\arcsin\left(-\frac{\sqrt{2}}{2}\right)$

$\frac{7\pi}{4}$ rad

c) $\sin^{-1} 3$ UND

NOT IN DOMAIN

Example 2 Evaluate inverse cosine functions

a) $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

$\frac{3\pi}{4}$

b) $\cos^{-1} 0$ $\frac{\pi}{2}$

c) $\arccos(-2)$ UND NOT IN DOMAIN

Example 3 Evaluate inverse tangent functions

a) $\tan^{-1}\sqrt{3}$ $r: \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\frac{\pi}{6}$ $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$\frac{y}{x} = \frac{1}{2} \cdot \frac{2}{\sqrt{3}}$ X

$\frac{\pi}{3}$

$\frac{\pi}{3}$ $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$\frac{y}{x} = \frac{\sqrt{3}}{2} \cdot \frac{2}{1} = \sqrt{3}$ ✓

b) arctan 0 $(1, 0)$ or $(x, y)$

$\frac{y}{x} = \frac{0}{1} = 0$

$0$

HW: 1-14

**Find the exact value of each expression, if it exists.**
(Examples 1–3)

1. $\sin^{-1} 0$ $= 0$
2. $\arcsin \frac{\sqrt{3}}{2}$ $= \pi/3$
3. $\arcsin \frac{\sqrt{2}}{2}$ $= \frac{\pi}{4}$
4. $\sin^{-1} \frac{1}{2}$ $= \pi/6$
5. $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ $= \frac{7\pi}{4}$
6. $\arccos 0$ $= \pi/2$
7. $\cos^{-1} \frac{\sqrt{2}}{2}$ $= \frac{\pi}{4}$
8. $\arccos(-1)$ $= \pi$
9. $\arccos \frac{\sqrt{3}}{2}$ $= \frac{\pi}{6}$
10. $\cos^{-1} \frac{1}{2}$ $= \pi/3$
11. $\arctan 1$ $= \pi/4$
12. $\arctan(-\sqrt{3})$ $= 5\pi/3$
13. $\tan^{-1} \frac{\sqrt{3}}{3}$ $= \pi/6$
14. $\tan^{-1} 0$ $= 0$

13 $\frac{1}{2} \cdot \frac{2}{\sqrt{3}} = \frac{\sqrt{3}}{3} = \frac{\pi}{6}$