T7 - The Law of Sines

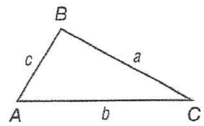
Solving Oblique Triangles using the Law of Sines

In order to solve these triangles you need 3 pieces of info falling into one of these conditions:

- AAS; the measure of two angles and a non-included side
- ASA; the measure of two angles and the included side, OR
- SSA; the measure of two sides and a non-included angle

Key Concept **Law of Sines**

If $\triangle ABC$ has side lengths a, b, and c representing the lengths of the sides opposite the angles with measures A, B, and C, then $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.

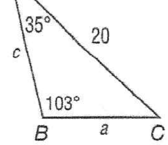


Example 1 Apply the Law of Sines (AAS)

Solve $\triangle ABC$. Round side lengths to the nearest tenth and angles to the nearest degree

① $\frac{\sin A}{a} = \frac{\sin B}{b} \Rightarrow \frac{\sin 35}{a} = \frac{\sin 103}{20} \Rightarrow 20 \sin 35 = a \sin 103°$

$a = \frac{20 \sin 35°}{\sin 103°} = 11.8$



② $C = 180 - (35 + 103)$

$= 42°$

$A = 35°$ $\boxed{a = 11.8}$

$B = 103°$ $b = 20$

$\boxed{C = 42°}$ $\boxed{c = 13.7}$

③ $\frac{\sin B}{b} = \frac{\sin C}{c} \Rightarrow \frac{\sin 103}{20} = \frac{\sin 42}{c} \Rightarrow 20 \sin 42 = c \sin 103$

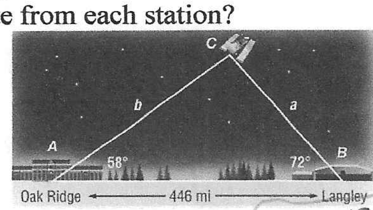
$c = \frac{20 \sin 42°}{\sin 103°} = 13.7$

Example 2 Apply the Law of Sines (ASA)

An Earth orbiting satellite is passing between the Oak Ridge Laboratory in Tennessee and the Langley Research Center in Virginia, which are 446 miles apart. If the angles of elevation to the satellite from the two facilities are 58° and 72°, respectively, how far is the satellite from each station?

① $C = 180 - (58 + 72) = 50°$

② $\frac{\sin A}{a} = \frac{\sin C}{c} \Rightarrow \frac{\sin 58}{a} = \frac{\sin 50}{446} \Rightarrow a = 494$



$A = 58$ $\boxed{a = 494 \text{ mi}}$

$B = 72$ $\boxed{b = 554 \text{ mi}}$

$\boxed{C = 50}$ $c = 446$

③ $\frac{\sin B}{b} = \frac{\sin C}{c} \Rightarrow \frac{\sin 72}{b} = \frac{\sin 50}{446} \Rightarrow b = 554$

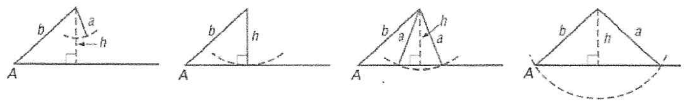
a) $A \geq 90$ ✓
$a > b$ one solution

b) $h = b\sin A$
$= 11\sin 61^\circ$
$= 9.621$
$h > a$
No solution

Key Concept **The Ambiguous Case (SSA)**

Consider a triangle in which a, b, and A are given. For the acute case, $\sin A = \frac{h}{b}$, so $h = b \sin A$.

A is Acute. ($A < 90°$)



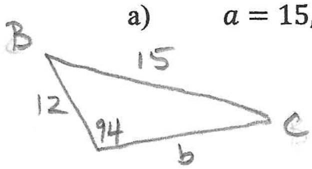
$a < b$ and $a < h$	$a < b$ and $a = h$	$a < b$ and $a > h$	$a \geq b$
no solution	one solution	two solutions	one solution

A is Right or Obtuse. ($A \geq 90°$)

$a \leq b$, no solution　　　　$a > b$, one solution

Example 3　　SSA – one or no solutions

a)　$a = 15, c = 12, A = 94°$



$A = 94^\circ$	$a = 15$
$B = 33^\circ$	$b = 8.2$
$C = 53^\circ$	$c = 12$

① $\frac{\sin A}{a} = \frac{\sin C}{c}$

$\frac{\sin 94}{15} = \frac{\sin C}{12}$

$\sin C = \frac{12\sin 94}{15}$

$C = \sin^{-1}\left(\frac{12\sin 95}{15}\right)$

$C = 53^\circ$

② $B = 180 - (94 + 53)$

③ $\frac{\sin B}{b} = \frac{\sin A}{a}$

$\frac{\sin 33}{b} = \frac{\sin 94}{15}$

$b = \frac{15\sin 33}{\sin 94}$

$b = 8.2$

b)　$a = 9, b = 11, A = 61°$

$A = 61$	$a = 9$
$B =$	$b = 11$
$C =$	$c =$

① $\frac{\sin A}{a} = \frac{\sin B}{b} \Rightarrow \frac{\sin 61}{9} = \frac{\sin B}{11}$

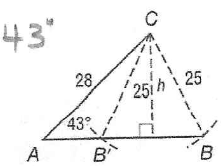
$B = \sin^{-1}\left(\frac{11\sin 61}{9}\right)$

$B =$

Example 4　　SSA – Two solutions

$A = 43°, a = 25, b = 28$

$h = 28\sin 43^\circ$
$h = 19$
$a < b$
$h < a \rightarrow 2$ △'s



△ ONE

$A = 43^\circ$	$a = 25$
$B_1 = 50^\circ$	$b = 28$
$C_1 = 87^\circ$	$c_1 = 36.6$

△ TWO

$A = 43^\circ$	$a = 25$
$B_2 = 130^\circ$	$b = 28$
$C_2 = 7^\circ$	$c_2 = 4.5$

① $\frac{\sin B_1}{28} = \frac{\sin 43}{25}$

$B_1 = \sin^{-1}\left(\frac{28\sin 43}{25}\right) = 50^\circ$

② $C_1 = 180 - (43 + 50) = 87^\circ$

③ $\frac{\sin A}{a} = \frac{\sin C}{c}$

$\frac{\sin 43}{25} = \frac{\sin 87}{c} \Rightarrow c_1 = \frac{25\sin 87}{\sin 43} = 36.6$

④ $B_2 = 180 - B_1$
$= 180 - 50$
$= 130^\circ$

⑤ $C_2 = 180 - (130 + 43)$
$= 7^\circ$

⑥ $\frac{\sin A}{a} = \frac{\sin C_2}{c_2}$

$\frac{\sin 43}{25} = \frac{\sin 7}{C_2}$

$C_2 = \frac{25\sin 7^\circ}{\sin 43} = 4.5$

HW: (1-4), (10-13), (19-22)

Solve each triangle. Round to the nearest tenth, if necessary. (Examples 1 and 2)

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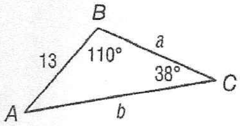

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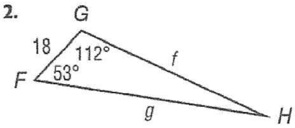

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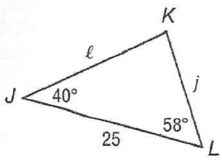

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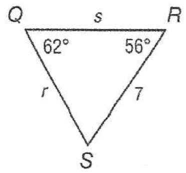

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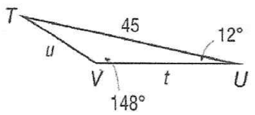

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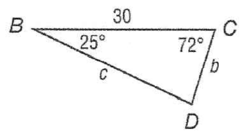
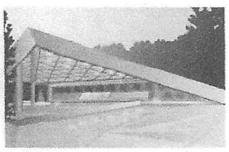


7. **GOLF** A golfer misses a 12-foot putt by putting 3° off course. The hole now lies at a 129° angle between the ball and its spot before the putt. What distance does the golfer need to putt in order to make the shot? (Examples 1 and 2)

8. **ARCHITECTURE** An architect's client wants to build a home based on the architect Jon Lautner's Sheats-Goldstein House. The length of the patio will be 60 feet. The left side of the roof will be at a 49° angle of elevation, and the right side will be at an 18° angle of elevation. Determine the lengths of the left and right sides of the roof and the angle at which they will meet. (Examples 1 and 2)



9. **TRAVEL** For the initial 90 miles of a flight, the pilot heads 8° off course in order to avoid a storm. The pilot then changes direction to head toward the destination for the remainder of the flight, making a 157° angle to the first flight course. (Examples 1 and 2)

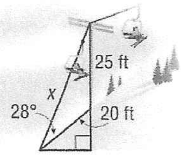
   a. Determine the total distance of the flight.

   b. Determine the distance of a direct flight to the destination.

Find all solutions for the given triangle, if possible. If no solution exists, write *no solution*. Round side lengths to the nearest tenth and angle measures to the nearest degree. (Example 3)

10. $a = 9, b = 7, A = 108°$
11. $a = 14, b = 15, A = 117°$
12. $a = 18, b = 12, A = 27°$
13. $a = 35, b = 24, A = 92°$
14. $a = 14, b = 6, A = 145°$
15. $a = 19, b = 38, A = 30°$
16. $a = 5, b = 6, A = 63°$
17. $a = 10, b = \sqrt{200}, A = 45°$

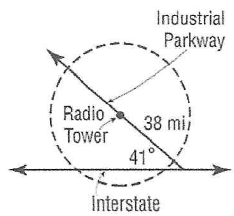
18. **SKIING** A ski lift rises at a 28° angle during the first 20 feet up a mountain to achieve a height of 25 feet, which is the height maintained during the remainder of the ride up the mountain. Determine the length of cable needed for this initial rise. (Example 3)



Find two triangles with the given angle measure and side lengths. Round side lengths to the nearest tenth and angle measures to the nearest degree. (Example 4)

19. $A = 39°, a = 12, b = 17$
20. $A = 26°, a = 5, b = 9$
21. $A = 61°, a = 14, b = 15$
22. $A = 47°, a = 25, b = 34$
23. $A = 54°, a = 31, b = 36$
24. $A = 18°, a = 8, b = 13$

25. **BROADCASTING** A radio tower located 38 miles along Industrial Parkway transmits radio broadcasts over a 30-mile radius. Industrial Parkway intersects the interstate at a 41° angle. How far along the interstate can vehicles pick up the broadcasting signal? (Example 4)



26. **BOATING** The light from a lighthouse can be seen from an 18-mile radius. A boat is anchored so that it can just see the light from the lighthouse. A second boat is located 25 miles from the lighthouse and is headed straight toward it, making a 44° angle with the lighthouse and the first boat. Find the distance between the two boats when the second boat enters the radius of the lighthouse light. (Example 4)

Solve each triangle. Round side lengths to the nearest tenth and angle measures to the nearest degree. (Examples 5 and 6)

27. $\triangle ABC$, if $A = 42°$, $b = 12$, and $c = 19$
28. $\triangle XYZ$, if $x = 5$, $y = 18$, and $z = 14$
29. $\triangle PQR$, if $P = 73°$, $q = 7$, and $r = 15$
30. $\triangle JKL$, if $J = 125°$, $k = 24$, and $l = 33$
31. $\triangle RST$, if $r = 35$, $s = 22$, and $t = 25$
32. $\triangle FGH$, if $f = 39$, $g = 50$, and $h = 64$
33. $\triangle BCD$, if $B = 16°$, $c = 27$, and $d = 3$
34. $\triangle LMN$, if $\ell = 12$, $m = 4$, and $n = 9$