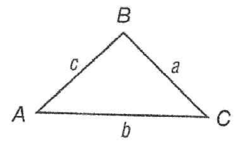
T8 - The Law of Cosines

Key Concept **Law of Cosines**

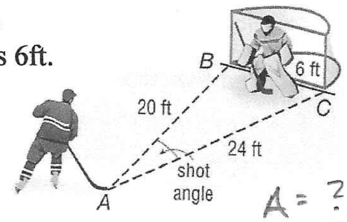
In $\triangle ABC$, if sides with lengths a, b, and c are opposite angles with measures A, B, and C, respectively, then the following are true.



$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$b^2 = a^2 + c^2 - 2ac \cos B$$
$$c^2 = a^2 + b^2 - 2ab \cos C$$

Example 5 Apply the Law of Cosines (SSS)

When a hockey player attempts a shot, he is 20ft. from the left post of the goal and 24ft. from the right post, as shown. If a regulation hockey goal is 6ft. wide, what is the player's shot angle to the nearest degree?



$A = ?$ $a = 6$
$B =$ $b = 24$
$C =$ $c = 20$

$a^2 = b^2 + c^2 - 2bc \cos A$

$6^2 = 24^2 + 20^2 - 2(24)(20) \cos A$

$36 = 576 + 400 - 960 \cos A$

$36 = 976 - 960 \cos A$

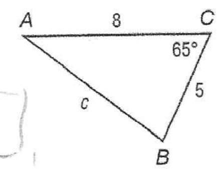
$-940 = -960 \cos A$

$\cos A = \frac{940}{960}$ $A = \arccos\left(\frac{946}{960}\right) = 11.7 = 12^\circ$

Example 6 Apply the Law of Cosines (SAS)

Solve ΔABC. Round side lengths to the nearest tenth and angle measures to the nearest degree.

$A = 38^\circ$ $a = 5$
$B = 77^\circ$ $b = 8$
$C = 65^\circ$ $c = 7.4$



① $c^2 = a^2 + b^2 - 2ab \cos C$

$= 25 + 64 - 80 \cos 65^\circ$

$= 89 - (80 \cos 65)$

$c = \sqrt{89 - (80 \cos 65)}$

$c = 7.4$

③ $B = 180 - (38 + 65)$

$B = 77^\circ$

② $\frac{\sin A}{a} = \frac{\sin C}{c}$

$\frac{\sin A}{5} = \frac{\sin 65}{7.4}$

$A = \sin^{-1}\left(\frac{5 \sin 65}{7.4}\right)$

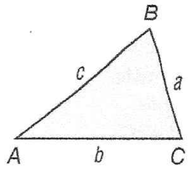
$A = 38^\circ$

Key Concept **Heron's Formula**

If the measures of the sides of $\triangle ABC$ are a, b, and c, then the area of the triangle is

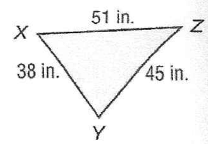
$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)},$$

where $s = \frac{1}{2}(a+b+c)$.



Example 7 Heron's Formula

Find the area of ΔXYZ.



① $s = \frac{1}{2}(x+y+z)$

$= \frac{1}{2}(45+51+38)$

$s = \frac{1}{2}(134)$

$s = 67.5$

② $A = \sqrt{67(67-45)(67-51)(67-38)}$

$= \sqrt{67(22)(16)(29)}$

$= \sqrt{683936}$

$A = 827 \text{ in}^2$

Solve each triangle. Round side lengths to the nearest tenth and angle measures to the nearest degree. (Examples 5 and 6)

27. $\triangle ABC$, if $A = 42°$, $b = 12$, and $c = 19$

28. $\triangle XYZ$, if $x = 5$, $y = 18$, and $z = 14$

29. $\triangle PQR$, if $P = 73°$, $q = 7$, and $r = 15$

30. $\triangle JKL$, if $J = 125°$, $k = 24$, and $l = 33$

31. $\triangle RST$, if $r = 35$, $s = 22$, and $t = 25$

32. $\triangle FGH$, if $f = 39$, $g = 50$, and $h = 64$

33. $\triangle BCD$, if $B = 16°$, $c = 27$, and $d = 3$

34. $\triangle LMN$, if $\ell = 12$, $m = 4$, and $n = 9$

Use Heron's Formula to find the area of each triangle. Round to the nearest tenth. (Example 7)

37. $x = 9$ cm, $y = 11$ cm, $z = 16$ cm

38. $x = 29$ in., $y = 25$ in., $z = 27$ in.

39. $x = 58$ ft, $y = 40$ ft, $z = 63$ ft

40. $x = 37$ mm, $y = 10$ mm, $z = 34$ mm

41. $x = 8$ yd, $y = 15$ yd, $z = 8$ yd

42. $x = 133$ mi, $y = 82$ mi, $z = 77$ mi